Solving Equations and Differentiation

Solving a function or an expression algebraically

You can set an expression or function equal to another expression, function, or number inside a `solve` command.As an example, you may want to find where the following two parabolas intersect.

```
> g := 9*x^2-14;
> h:=-x^2;
> plot([g,h],x=-2..2);
> solve(g=h,x);
```

The plot shows that there are two intersection points and the `solve` command finds both x values. It is good to get into the habit of naming your output so you can use it in a later command. Giving the x values a name makes it easy to plug them into the expression to find the y values.

```
> ip:=solve(g=h,x);
```

Since there are two x values called ip, use [ ] to call up the one you want.

```
> subs(x=ip[1],g);
> subs(x=ip[2],h);
```

Therefore the two intersection points are $(\frac{\sqrt{35}}{5}, \frac{-7}{5})$ and $(\frac{-\sqrt{35}}{5}, \frac{-7}{5})$. This seems like the answer shown on the graph.

Solving a function or an expression numerically

If you want to find where the following function crosses the x-axis, just set it equal to zero.

```
> f:=theta->-1/2*theta+sin(theta);
> plot(f(theta),theta=-8*Pi..8*Pi);
> solve(f(theta)=0,theta);
```

Wow, what is that?!?! We know from the graph that there should be three answers and `solve` wasn't a great option so try `fsolve`.

```
> fsolve(f(theta)=0,theta);
```

Where are the other two answers!? This is actually how `fsolve` usually works. It shoots for one answer and only gives that one. But you can tell `fsolve` where to look by getting an idea from the graph and typing that domain into the `fsolve` command.

```
> a:=fsolve(f(theta)=0,theta=-5..-1);
> b:=fsolve(f(theta)=0,theta=-1..1);
> c:=fsolve(f(theta)=0,theta=1..5);
```

To find the y values just plug in the names of the x values.

```
> f(a);
> f(b);
> f(c);
```

(Of course the y-values are zero!)

The Derivative

The Limit Definition of the Derivative

The limit definition of the derivative of $f(x)$ often written as $f'(x)$ is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

It can be interpreted geometrically as the slope of the tangent line to the graph of $f(x)$ at a point $x = a$ and functionally as the instantaneous rate of change of f at $x = a$. You can use the definition and the Maple limit command to compute derivatives directly, as shown below. You can also compute derivatives using Maple's `diff` or `D` command. The following limit determines f'(x).

```
> limit((f(x+h)-f(x))/h,h=0);
```

The example below shows how to use the limit definition of derivative to find $f'(1)$ with Maple.

```
> f := x -> x^2+3*x+5;
> limit((f(1+h)-f(1))/h,h=0);
```

The Maple D and diff commands

These commands can be summarized as follows.

- The `D` command acts on a function.
- The `diff` command acts on an expression or a function and differentiates that expression with respect to a variable specified by the user.

When you use the `D` operator to compute the derivative of a function, be careful with the parentheses. It is one of the only commands in Maple where the f gets its own parentheses.

```
> f:=x->x^2;
> D(f)(x);
```

Finding the derivative at a specific x value is easy. (Again be careful of the parentheses.)

```
> D(f)(2);
```

The `D` operator **CANNOT** be used on expressions. To differentiate expressions, you need to use the `diff` command. Here is an example.

```
> p:=3*x+2;
> diff(p,x);
```

Remember the `diff` command can also be applied to functions. However, the syntax for plugging in an x value is a little longer with the `diff` command. To compute the value of the derivative at a specific value of x requires you to use the `subs` command. First, give the `diff` command a name so you can call it up in the `subs` command.

```
> pprime:=diff(p,x);
> subs(x=2,pprime);
```

Another option is to embed the commands.

```
>subs(x=2,diff(p,x));
```

Exercises

1. For the functions $f(x) = \ln(x)\cos(x-1)$ and $g(x) = \exp((x-2)/12) + 0.1$, plot both functions on the same graph using an x-domain that clearly shows all the intersection points and then find the x and y coordinates of the intersection points using Maple's solving capabilities.

2. Find the dervative of the function $f(x) = \dfrac{(x^2-3)^2}{x^4+x^2+1}$ using the limit definition of the derivative, the `diff` command and then the `D` command and then use all three methods to find the slope of f at $x = -5$.

3. For the function in the last exercise, find all points on the graph of $f(x)$ where the tangent line is horizontal. Remember that a point has an x and a y value.