Solving Equations and Differentiation

Solving a function or an expression algebraically

You can set an expression or function equal to another expression, function, or number inside a solve command. As an example, you may want to find where the following two parabolas intersect.

```plaintext
> g := 9*x^2-14;
> h:=-x^2;
> plot([g,h],x=-2..2);
> solve(g=h,x);
```

The plot shows that there are two intersection points and the `solve` command finds both x values. It is good to get into the habit of naming your output so you can use it in a later command. Giving the x values a name makes it easy to plug them into the expression to find the y values.

```plaintext
> ip:=solve(g=h,x);
```

Since there are two x values called `ip`, use `[ ]` to call up the one you want.

```plaintext
> subs(x=ip[1],g);
> subs(x=ip[2],h);
```

Therefore the two intersection points are \((\sqrt{\frac{35}{5}}, -\frac{7}{5})\) and \((-\sqrt{\frac{35}{5}}, -\frac{7}{5})\). This seems like the answer shown on the graph.

Solving a function or an expression numerically

If you want to find where the following function crosses the x-axis, just set it equal to zero.

```plaintext
> f:=theta->-1/2*theta+sin(theta);
> plot(f(theta),theta=-8*Pi..8*Pi);
> solve(f(theta)=0,theta);
```

Wow, what is that?!?! We know from the graph that there should be three answers and `solve` wasn’t a great option so try `fsolve`.

```plaintext
> fsolve(f(theta)=0,theta);
```

Where are the other two answers?! This is actually how `fsolve` usually works. It shoots for one answer and only gives that one. But you can tell `fsolve` where to look by getting an idea from the graph and typing that domain into the `fsolve` command.

```plaintext
> a:=fsolve(f(theta)=0,theta=-5..-1);
> b:=fsolve(f(theta)=0,theta=-1..1);
> c:=fsolve(f(theta)=0,theta=1..5);
```
To find the \( y \) values just plug in the names of the \( x \) values.

\[
> f(a);
> f(b);
> f(c);
\]

(Of course the \( y \)-values are zero!)

**The Derivative**

**The Limit Definition of the Derivative**

The limit definition of the derivative of \( f(x) \) often written as \( f'(x) \) is defined as:

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

It can be interpreted geometrically as the slope of the tangent line to the graph of \( f(x) \) at a point \( x = a \) and functionally as the instantaneous rate of change of \( f \) at \( x = a \). You can use the definition and the Maple limit command to compute derivatives directly, as shown below. You can also compute derivatives using Maple’s `diff` or `D` command. The following limit determines \( f'(x) \).

\[
> \text{limit}((f(x+h)-f(x))/h, h=0);
\]

The example below shows how to use the limit definition of derivative to find \( f'(1) \) with Maple.

\[
> f := x \rightarrow x^2+3*x+5;
> \text{limit}((f(1+h)-f(1))/h, h=0);
\]

**The Maple D and diff commands**

These commands can be summarized as follows.

- The `D` command acts on a function.

- The `diff` command acts on an expression or a function and differentiates that expression with respect to a variable specified by the user.

When you use the `D` operator to compute the derivative of a function, be careful with the parentheses. It is one of the only commands in Maple where the \( f \) gets its own parentheses.

\[
> f:=x->x^2;
> D(f)(x);
\]

Finding the derivative at a specific \( x \) value is easy. (Again be careful of the parentheses.)

\[
> D(f)(2);
\]
The D operator **CANNOT** be used on expressions. To differentiate expressions, you need to use the **diff** command. Here is an example.

> p:=3*x+2;
> diff(p,x);

Remember the **diff** command can also be applied to functions. However, the syntax for plugging in an x value is a little longer with the **diff** command. To compute the value of the derivative at a specific value of x requires you to use the **subs** command. First, give the **diff** command a name so you can call it up in the **subs** command.

> pprime:=diff(p,x);
> subs(x=2,pprime);

Another option is to embed the commands.

> subs(x=2,diff(p,x));

**Exercises**

1. For the functions \( f(x) = \ln(x) \cos(x - 1) \) and \( g(x) = \exp((x - 2)/12) + 0.1 \), plot both functions on the same graph using an x-domain that clearly shows all the intersection points and then find the x and y coordinates of the intersection points using Maple’s solving capabilities.

2. Find the derivative of the function \( f(x) = \frac{(x^2 - 3)^2}{x^4 + x^2 + 1} \) using the limit definition of the derivative, the **diff** command and then the D command and then use all three methods to find the slope of f at \( x = -5 \).

3. For the function in the last exercise, find all points on the graph of \( f(x) \) where the tangent line is horizontal. Remember that a point has an x and a y value.