

1 Maple Introduction

1.1 Getting Started

The software package Maple is an example of a *Computer Algebra System* (CAS for short), meaning that it is capable of dealing with problems in symbolic form. This is in contrast to the numerical approach that is usually involved in programming computers to solve problems. A CAS is capable of understanding functions and expressions, but can also be used to perform algebraic manipulations with expressions and functions. For example, if you give a CAS an expression like $x^2 + 2 * x + 1$ it understands that x is a variable and Maple can do things like evaluate the expression at a given value of x , factor the expression, substitute another expression in for x , multiply two expressions together, and more - all without making mistakes and much more rapidly than you can do them by hand. Maple also will let you define your own functions, which can be plotted, manipulated, combined with other functions, etc. Maple will also perform many of the manipulations that usually occupy a major part of a calculus course - differentiation, integration, substitution, solving differential equations, etc.

Maple is a powerful tool that can free you from a lot of calculational drudgery, but there is a catch. To use it properly, you have to understand the rules of the game. These rules can be divided into two parts: rules for Maple syntax, i.e. what are the commands that Maple understands and how you must express them, and rules of mathematics. Maple can perform calculations at blinding speed, but you are responsible for making those calculations meaningful and mathematically correct. In this section, we will concentrate on the rules for Maple syntax.

1.2 Maple Commands

To get Maple to do computations for you, you have to type in commands. In a Maple worksheet, you can only type in commands at a input line, which begins with the input prompt, `[>]`.

To type and issue a command in Maple, first use the mouse to move the cursor to an empty input line and click the left mouse button to select that line for typing. Then type in the command and end it with a semicolon (;). To issue the command, press the return key. The output of the command will appear directly below the input line. This region where output appears is called a Maple Output Region.

If the output of the command is not what you wanted, perhaps due to a typing error, you can use the mouse to move the cursor back to the input line and correct the command. Use the arrow keys to move around in the line and the delete or backspace key to delete unwanted characters. Once you have corrected the command, you can press the return key to issue the command. You don't have to move the cursor to the end of the line first.

When you go back and change a command that has already been issued, the old result is discarded and replaced by the new one.

1.2.1 Entering an expression

Maple understands common mathematical notation in defining expressions, as shown in Table 1. Note that Maple doesn't always allow implied multiplication.

Operator	Meaning
+	addition
-	subtraction
*	multiplication
**	exponentiation
^	exponentiation

Table 1: Binary arithmetic operators

An expression such as $x^3 + 4x^2 - 9$ can be entered as follows.

```
[> r:= x^3 + 4*x^2-9;
```

$$r := x^3 + 4x^2 - 9$$

```
[> s:=sin(x) *exp(x);
```

$$s := \sin(x) e^x$$

```
[> t:= 2*x+y;
```

$$t := 2x + y$$

```
[> u:=cos(x)*y^2;
```

$$u := \cos(x) y^2$$

The name before the colon and equals sign ($:=$) is not really part of the expression, but is a name or label for the expression. It is best to name or label expressions so that you can actually use them later on. Note that the colon before the '=' is necessary. If you leave out the colon, you will not get an error, but you will have entered an equation, not a labeled expression. Furthermore, your equation won't have a label and so you won't be able to refer to it. Also, there must not be a space between the colon and the equals sign. Later on you will see examples of how labels are useful.

Every line you enter, with the exception of help commands, must have either a colon or semicolon at the end. Without a colon or semicolon, Maple will not process the command. (A colon is used if you do not want to see the output of a command. Use a semi-colon if you want to see the output.)

1.2.2 Entering a function

Maple makes a strong distinction (as it should) between a labeled expression and a function. A labeled expression is made up of variables, numbers, and the standard mathematical operations of addition, subtraction, multiplication, division, and exponentiation along with a label . For example,

```
[> expr1:=x^(12)-4/x ;
```

defines the expression $x^{12} - 4/x$ and labels it with `expr1`.

A function, on the other hand, is a more abstract object. It has a name or label, but it also has a set of allowed input values and a rule for determining exactly one output value from each input value. That is, a function is an input-output machine. In practice, expressions are very often used to define functions, for example $f(x) = 2x$. Later on, you will see examples that will help you understand the difference between a function and a labeled expression in Maple.

The syntax for defining functions in Maple uses the symbol “->”, made by typing the two characters - and >.

Suppose you want to enter the function $f(x) = x^3 + 2x^2 - \frac{1}{x}$. In Maple you would type:

```
[> f := x -> x^3+2*x^2-1/x;
```

$$f := x \rightarrow x^3 + 2x^2 - \frac{1}{x}$$

Note that the colon before the '=' is necessary. The `f` before the colon is a label like we used before that will let you refer to the function in a subsequent command. Here are a few more examples of defining functions.

```
[> g := x -> cos(x + 1);
```

$$g := x \rightarrow \cos(x + 1)$$

```
[> h := x -> exp(x) + ln(2*x + 1);
```

$$h := x \rightarrow e^x + \ln(2x + 1)$$

Creating a function of more than one variable is very similar to creating functions of one variable. For example:

```
[> j := (x,y) -> x^2 + y^2;
```

$$j := (x, y) \rightarrow x^2 + y^2$$

```
[> k := (x,y,z) -> x - 5*y + cos(z*x);
```

$$k := (x, y, z) \rightarrow x - 5y + \cos(zx)$$

1.2.3 Evaluating functions and expressions

In order to evaluate an expression at a given value of x , you must use the `subs` command. For example,

```
[> subs(x=2,r);
```

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evaluates r (defined earlier in the handout) at $x = 2$. Here are a few more examples of evaluating expressions:

```
[> subs(x=a,s);
```

$$\sin(a) e^a$$

[> subs(x=a+h,r);

$$(a+h)^3 + 4(a+h)^2 - 9$$

[> subs(x=1,y=2,t);

$$4$$

In Maple, functions are much easier to evaluate than expressions. In order to evaluate the function f (defined earlier in the handout) at $x = 5$, you need only type

[> f(5);

$$\frac{874}{5}$$

These are a few more examples of evaluating functions.

[> g(a+2);

$$\cos(a+3)$$

[> h(x);

$$e^x + \ln(2x+1)$$

[> j(1,2);

$$5$$

1.2.4 More on Functions

We have already seen how to evaluate a function at a specific value of x , but there is more you can do with functions. You can also compose functions or define new functions in terms of old ones. Some examples are given below. Note that the independent variable you use to define the function is only a placeholder. That is, if you define a function f using x as the independent variable, Maple will cope with changing the independent variable in a way that is mathematically correct.

[> f:= x -> x^2+1;

$$f := x \rightarrow x^2 + 1$$

[> f(2);

$$5$$

[> f(3*t);

$$9t^2 + 1$$

[> h := t -> sin(2*t);

$$h := t \rightarrow \sin(2t)$$

```
[> h(0);
```

$$0$$

```
[> h(x);
```

$$\sin(2x)$$

```
[> f(h(t));
```

$$\sin(2t)^2 + 1$$

```
[> f(h(x));
```

$$\sin(2x)^2 + 1$$

1.3 Operations

1.3.1 Differentiation

Maple has two commands to find derivatives. For now we will only concern ourselves with the `diff` command. In order to take the derivative of the expression r (from above), type the following.

```
[> diff(r,x);
```

$$3x^2 + 8x$$

Notice that you do not have to name an expression and then differentiate. You can type the expression in the `diff` statement as shown below.

```
[> diff(x^3 + 4*x^2 -9,x);
```

$$3x^2 + 8x$$

The first argument in the parentheses is the expression or name of the expression. The second argument gives the variable which the derivative should be taken with respect to. Second derivatives can also be found with this command. To take the second derivative of r with respect to x , type the following command.

```
[> diff(r,x,x);
```

$$6x + 8$$

You can also assign names to these expressions.

```
[> r1:=diff(r,x);
```

$$r1 := 3x^2 + 8x$$

```
[> subs(x=1,r1);
```

The benefit of assigning a name to the derivative is now you can easily evaluate the derivative for values of x using the `subs` command.

Finding the derivative of a function is very similar. Suppose we are attempting to find the derivative of the function f from above. The following command is necessary.

```
[> diff(f(x),x);
```

$$3x^2 + 4x + \frac{1}{x^2}$$

What is the difference between the command for differentiating a function and the command for differentiating an expression?

1.3.2 Integration

The Maple command for integration is `int`. The syntax is similar to that of the `diff` command. To find the indefinite integral of the expression r , type the following:

```
[> int(r,x);
```

$$\frac{1}{4}x^4 + \frac{4}{3}x^3 - 9x$$

In order to find a definite integral of the expression r , the only modification needed is to add the limits of integration to the command. The command for $\int_1^3 x^3 + 4x^2 - 9$ follows.

```
[> int(r,x=1..3);
```

$$\frac{110}{3}$$

If it is impossible to compute the indefinite integral, Maple will return the integral unevaluated. Although it may be impossible for Maple to find an indefinite integral, using the `evalf` command Maple can use numerical methods to solve a definite integral.

```
[> int(sec(x^2),x);
```

$$\int \sec(x^2) dx$$

```
[> evalf(int(sec(x^2),x=0..1));
```

$$1.132697500$$

Integrating a function is similar to integrating an expression. To integrate the function f , you must use the following format.

```
[> int(f(x),x);
```

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 - \ln(x)$$

```
[> int(f(x),x=2..5);
```

$$\frac{921}{4} - \ln(5) + \ln(2)$$

1.4 Graphing

1.4.1 Two dimensional graphs

Maple will easily graph (plot) a given expression or function of one variable. The necessary command is `plot` and is best demonstrated through the following examples. Recall we have previously defined the expression $r = x^3 + 4 * x^2 - 9$ and the function $g(x) = \cos(x + 1)$.

```
[> plot(r,x=-5..5);
```

```
[> plot(g(x),x=0..2*Pi);
```

```
[> plot([g(x),r],x=-2..2);
```

```
[> plot([g(x),r,x^2+13],x=-2..2);
```

To plot more than one graph on a single coordinate system, you must enclose all functions and expressions within square braces `[]`. The `plot` command always needs a domain specified. If necessary, you can also specify a range. Compare the following two plots. In the second, the axes are closer to the same scale.

```
[> plot(g(x),x=0..2*Pi);
```

```
[> plot(g(x),x=0..2*Pi,y=-Pi..Pi);
```

1.4.2 Three dimensional graphs

Plotting 3-dimensional expressions and functions is similar to plotting 2-dimensional expressions and functions. Recall that we have previously defined an expression $t = 2 * x + y$ and a function $j(x, y) = x^2 + y^2$.

```
[> plot3d(t,x=-2..2,y=-2..2);
```

```
[> plot3d(j(x,y),x=-1..1,y=-1..1);
```

For a three dimensional function, both x and y intervals must be specified. You are allowed to also specify a range on the third axis as shown below. It is also possible to plot more than one three dimensional graph on a single coordinate axis. Once again, this is done by enclosing all functions within curly braces.

```
[> plot3d([j(x,y),t],x=-1..1,y=-1..1);
```

```
[> plot3d(sin(x)*cos(y),1,x=-2..2,y=-2..2);
```

1.5 Saving your work

To save your Maple worksheet, select the **Save** item from the **File** menu. This will pop up a file selection dialog. Type in a name for the file at the end of the string in the **Selection** window and click the **Save** button. The name of your worksheet is now shown as part of the title of your Maple window.

If you make changes in your worksheet and then choose the **Save** item again, the changes will automatically be saved to the same file you chose above. The file selection dialog only appears the first time you save a worksheet.

Saving your worksheet often while you are working on it is strongly encouraged. **You should save the worksheet every 5-10 minutes or so, just in case a disaster happens.**