Solving Equations with Maple

Introduction
The purpose of this lab is to introduce you to the basic commands needed in any Maple lab. There are

- 2 ways to enter a mathematical expression in Maple.
- 3 ways to plug in an x-value to get the y-value.
- 2 ways to find an x-value in an equation algebraically.

Pay close attention in this lab to all the variations in the syntax.

Entering a mathematical expression
Expressions such as \( x^3 + 3x^2 - x + 1 \) can be entered in a similar way to variable assignment. Choose a variable name to represent the expression and assign the expression to the variable as follows.

\[
> qq := x^3 + 3*x^2 - x + 1;
\]

Entering a function
Suppose you want to enter an expression as a function of x. In Maple you would type the following.

\[
> f := x -> x^2 + x^2 - 6;
\]

Entering an expression and a function differ as do how they are called up in a Maple command. The function must have the \((x)\) and the expression does not.

\[
> plot([qq, f(x)], x = -6..4);
\]

Evaluating functions and expressions
In order to evaluate an expression at a given x-value, you can use the \texttt{subs} or \texttt{eval} command.

\[
> \text{subs}(x=2, qq);
> \text{eval}(qq, x=\pi);
\]

In Maple, functions are much easier to evaluate than expressions.

\[
> f(2);
\]
Solving a function or an expression algebraically

You can set an expression or function equal to another expression, function, or number inside a solve command. As an example, you may want to find where the following two parabolas intersect.

```plaintext
> g := 9*x^2-14;
> h := -x^2;
> plot([g,h],x=-2..2);
> solve(g=h,x);
```

The plot shows that there are two intersection points and the solve command finds both x values. It is good to get into the habit of naming your output so you can use it in a later command. Giving the x values a name makes it easy to plug them into the function to find the y values.

```plaintext
> ip := solve(g=h,x);
```

Since there are two x values called ip, use [ ] to call up the one you want.

```plaintext
> eval(g, x=ip[1]);
> subs(x=ip[2], g);
```

Therefore the two intersection points are \((\sqrt{35}/5, -7/5)\) and \((-\sqrt{35}/5, -7/5)\). This seems like the answer shown on the graph.

Solving a function or an expression numerically

If you want to find where the following function crosses the x-axis, just set it equal to zero.

```plaintext
> f := theta -> -1/2*theta + sin(theta);
> plot(f(theta), theta=-3*Pi..3*Pi);
> solve(f(theta)=0, theta);
```

Wow, what is that?!?! We know from the graph that there should be three answers and solve wasn’t a great option so try fsolve.

```plaintext
> fsolve(f(theta)=0, theta);
```

Where are the other two answers!? This is actually how fsolve usually works. It shoots for one answer and only gives that one. But you can tell fsolve where to look by getting an idea from the graph and typing that domain into the fsolve command.

```plaintext
> a := fsolve(f(theta)=0, theta=-Pi..-1);
> b := fsolve(f(theta)=0, theta=-1..1);
> c := fsolve(f(theta)=0, theta=1..Pi);
```

To find the y values just plug in the names of the x values.

```plaintext
> f(a);
> f(b);
> f(c);
```

(Of course the y-values are zero!)
Exercises

1. Given the expressions $x^3 - 6x + 4$ and $-x + 4$ find the intersection points. (Do not change the answers to decimals)

2. Given the functions $f(x) = \sqrt{\frac{x}{2}} \sin(x)$ and $h(x) = e^{\frac{x}{12}} - \frac{11}{20}$ find the intersection points.