BSI Preparation Guide

This document describes the basic types of problems you are likely to encounter on the Basic Skills exam and provides examples to be worked out for each type.

Simple Integrals

These are integrals that can be done using only the most basic forms and properties of integrals. The basic forms can be found in the Elementary forms section of the Table of Integrals in your textbook and consist of numbers 2, 3, 4, 6, 7, and 16 and 17 with $a = 1$. See the examples below.

1. $\displaystyle\int_0^2 \left(x^4 + \pi\right)\, dx$
2. $\displaystyle\int (x^5 + 3x)\, dx$
3. $\displaystyle\int (x + \frac{2}{x})\, dx$

Integrals of $f(kx)$

These integrals are done using substitution, but they occur so frequently in applications that they rate their own section. The pattern for indefinite integrals of this type is shown below.

$$\int f(kx)\, dx = \frac{1}{k}\int f(u)\, du$$

For definite integrals, transforming the limits gives the following.

$$\int_a^b f(kx)\, dx = \frac{1}{k}\int_{ka}^{kb} f(u)\, du$$

Here are some examples.

4. $\displaystyle\int \cos(2x)\, dx$
5. $\displaystyle\int e^{3x}\, dx$
6. $\displaystyle\int e^{-x}\, dx$
7. $\displaystyle\int_0^{\pi} \sin(x/2)\, dx$

Inverse Trig functions

The two basic integrals are for arctan and arcsin. Variations on these are of the following two forms.

$$\int \frac{1}{c^2 + d^2x^2}\, dx = \frac{1}{cd}\int \frac{1}{1 + u^2}\, du$$

and

$$\int \frac{1}{\sqrt{c^2 - d^2x^2}}\, dx = \frac{1}{d}\int \frac{1}{\sqrt{1 - u^2}}\, du$$

Examples are shown below.

8. $\displaystyle\int \frac{1}{9 + 4x^2}\, dx$

9. $\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,dx$

10. $\displaystyle\int_0^{\frac{1}{3}} \frac{1}{1+9x^2}\,dx$

11. $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$

Substitution

If the integral is not of one of the types described above, then you should try to use substitution. The pattern for indefinite integrals is

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

and the following for definite integrals.

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Examples are shown below.

12. $\displaystyle\int_1^e \frac{\ln(2x)}{3x}\,dx$

13. $\displaystyle\int \frac{x}{\sqrt{x^2+4}}\,dx$

14. $\displaystyle\int x^3\sqrt{3+x^4}\,dx$

15. $\displaystyle\int \frac{2x}{1+x^4}\,dx$

16. $\displaystyle\int \cos^3(x)\sin(x)\,dx$

Integration by Parts

If the integral is not one of the types decribed above, then try integration by parts. See the examples below.

17. $\displaystyle\int xe^{-2x}\,dx$

18. $\displaystyle\int x\sin(x)\,dx$

19. $\displaystyle\int xe^{2x}\,dx$

20. $\displaystyle\int \ln(x)\,dx$

Fundamental Theorem of Calculus

The basic formula is the following.

$$\frac{d}{dx}\int_a^{g(x)} f(u)\,du = f(g(x)\,g'(x)$$

Examples appear below.

21. $\displaystyle\frac{d}{dx}\int_0^{x^2} \arcsin(2\pi\theta)\,d\theta$

22. $\displaystyle\frac{d}{dx}\int_0^{3x} \frac{2u}{1+u^2}\,du$

23. $\displaystyle\frac{d}{dx}\int_0^{\sin(x)} \frac{u+1}{1+e^u}\,du$