

Sample Final Exam - MA 1022 B '02

Work all of the problems. The use of calculators and notes is not allowed.

Part I - Basic Skills

Work the following problems and write your answers in the space provided.

1. $\int_0^1 (2x + 1)^5 dx$

ans. _____

2. $\int \frac{9}{9 + t^2} dt$

ans. _____

3. $\int \frac{\sin(\theta)}{9 + \cos^2(\theta)} d\theta$

ans. _____

4. $\int_{-1}^1 (x + 1)e^{-2x} dx$

ans. _____

5. $D_x [xe^{-x^2}]$

ans. _____

6. $\frac{dy}{dx}$ if $y = \arctan(3x)$

ans. _____

7. $D_x \int_x^{x^2} \arctan(3t) dt$

ans. _____

Part II

Work all of the following problems. Show your work in the space provided. You need not simplify your answers, but remember that on this part of the exam your work and your explanations are graded, not just the final answers.

8. Find the total area of the regions whose boundaries are

$$y = x^3, \quad y = x; \quad 0 \leq x \leq 2.$$

9. Consider the region R whose bounds are given by

$$y = \frac{1}{x}, \quad y = \sqrt{x}; \quad 1 \leq x \leq 4.$$

- (a) Find the volume generated by rotating R about the x -axis.
(b) Set up the integral for the volume generated when the given region R is rotated about the line $y = 5$.
DO NOT BOTHER TO EVALUATE THE INTEGRAL.

10. A large cylindrical tank is filled with maple syrup. The syrup weighs 40 pounds per cubic foot and the tank is 20 feet tall with a base radius of 10 feet. The syrup must be pumped out of the tank to a height of 50 feet above the top of the tank (where there is a large plate containing pancakes). Find the total work done in emptying the tank.
11. Find the centroid (center of mass) for a quarter circle in the first quadrant (x, y) -plane with center at $(0,0)$ and radius $r = 2$.

Note: You can simplify your work by using either Pappus' theorem or symmetry. In any case, you must explain your work clearly and justify your answer.

12. In this problem, you will evaluate the following integral:

$$\int_1^3 4x^2 dx$$

- (a) Construct a Riemann sum with $n = 4$ rectangles to approximate the integral. In particular, define Δx and the points \bar{x}_i where you evaluate the function. You may use a regular mesh (with equal subintervals) in your definition.
(b) Step up to the general case: What is the form of the Riemann sum for the example above with an arbitrary number n of subintervals?
(c) Compute the limit in the Riemann sum and evaluate the integral.

Note: You may need to use the fact that

$$\sum_{k=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

13. Use logarithmic differentiation to find the derivative of $y = x^{\sin(x)}$.
14. Suppose that the population P (in millions) of a colony of bacteria is modeled by the differential equation $P'(t) = 0.03P(t)$. If $P(2000) = 250$, find the formula for $P(t)$.