Rectangular Approximations for Area

Area Approximations

In good ole Geometry, the areas of some shapes are well known.

```
>plot([[[1, 0], [-6, 0], [-6, 5], [1, 5], [1, 0]], [[2, 0], [3, 5], [7, 0], [2, 0]],
 sqrt(25-(x-13)^2)], x = -7 .. 20, scaling = constrained, filled, axes = none);
```

But how about...

```
>plot(abs((1/13)*x^5*sin(x)+sin(x))+10000, x = 5 .. 18.85, y = 0 .. 140000, filled);
```

Using a known formula - the rectangle - approximating the area with this rectangle...

```
>plot([[[0, 0], [0, 133389.77], [18.85, 133389.77], [18.85, 0], [0, 0]],
abs((1/13)*x^5*sin(x)+sin(x))+10000], x = 5 .. 18.85, y = 0 .. 140000, filled)
```

would overestimate the area. Or this rectangle...

```
>plot([[[0, 0], [0, 10000], [18.85, 10000], [18.85, 0], [0, 0]],
abs((1/13)*x^5*sin(x)+sin(x))+10000], x = 5 .. 18.85, y = 0 .. 140000, filled);
```

would underestimate the area. Maybe use more than one rectangle...

```
>with(student);
>leftbox(abs((1/13)*x^5*sin(x)+sin(x))+10000, x = 5 .. 18.85, 3);
>rightbox(abs((1/13)*x^5*sin(x)+sin(x))+10000, x = 5 .. 18.85, 3);
>middlebox(abs((1/13)*x^5*sin(x)+sin(x))+10000, x = 5 .. 18.85, 3);
```

Look at the three commands above. The `leftbox` draws the three rectangles with the height on the left of each touching the function. The `rightbox` command draws the three rectangles with the height on the right of each touching the function. The `middlebox` command draws the three rectangles with the height in the center of each touching the function. If you try more rectangles the area should get closer to the real area.

```
>middlebox(abs((1/13)*x^5*sin(x)+sin(x))+10000, x = 5 .. 18.85, 12);
```

Exercises

1. A) Plot $f(x) = 7x^3 + 14$ on the domain $-1 \leq x \leq 2$ using the right-end-point rule (`rightbox`) and the left-end-point rule (`leftbox`) with 5 rectangles. Which rule over estimates the real area and which under estimates the real area.

   B) Plot $g(x) = 20x - \frac{x^4}{2} + 100$ on the domain $3 \leq x \leq 4$ using the right-end-point rule (`rightbox`) and the left-end-point rule (`leftbox`) with 5 rectangles. Which rule over estimates the real area and which under estimates the real area.

C) Plot $h(x) = 1 - 7x$ on the domain $-5 \le x \le 0$ using the right-end-point rule (`rightbox`) and the left-end-point rule (`leftbox`) with 5 rectangles. Which rule over estimates the real area and which under estimates the real area.

D) Looking at your graphs, what determines whether the rule will over or under estimate the area. Can you state your answer using the first or second derivative of a function?

2. Plot the following functions using the right and left end-point rules using 10 rectangles on the given domains.

   A) $j(x) = \frac{3+x}{x^2}$ on the domain $1 \le x \le 5$

   B) $k(x) = \sqrt{9 - x^2}$ on the domain $0 \le x \le 3$

   C) $l(x) = e^{\frac{-(x^2)}{\pi}}$ on the domain $\frac{10\pi}{16} \le x \le \pi$

   D) All the functions are decreasing on the given intervals. Looking at the graphs, which rule will give an area approximation slightly closer to the real area. Why? State your answer using the first or second derivative of a function.

3. A) Using the function $l(x)$, use the midpoint rule (`middlebox`) with 10 rectangles to graph the rectangular approximation on the same domain. Then answer the following question: Of the three graphs of $l(x)$ which will give you the area closest to the real area under `l(x)`? Why?

   B) Can you come up with a (non-horizontal) function that `middlebox` will give the exact area of? Show the graph and explain.

   C) Can you come up with a (non-horizontal) function that `leftbox` or `rightbox` will give the exact area of? Show the graph and explain.