

# The Definite Integral

## Introduction

There are two main ways to think of the definite integral. The easiest one to understand is as a means for computing areas (and volumes). The second way the definite integral is used is as a sum. That is, we use the definite integral to “add things up”. Here are some examples.

- Computing net or total distance traveled by a moving object.
- Computing average values, e.g. centroids and centers of mass, moments of inertia, and averages of probability distributions.

This lab is intended to introduce you to Maple commands for computing integrals, including applications of integrals.

## Definite and indefinite integrals with Maple

The basic Maple command for computing definite and indefinite integrals is the `int` command.

To compute the indefinite integral

$$\int x^2 dx$$

with Maple:

```
> int(x^2,x);
```

Note that Maple does **not** include a constant of integration. Suppose you wanted to compute the following definite integral with Maple.

$$\int_0^4 x^2 dx$$

The command to use is:

```
> int(x^2,x=0..4);
```

## Exercises

1. Use Maple to compute the each of the following definite integrals:

A)  $\int_0^3 \frac{x^4 + 2x^2 + 1}{x^2 + 1} dx$

B)  $\int_{-3}^3 \frac{x^4 + 2x^2 + 1}{x^2 + 1} dx$

- C) Explain why your answer to this part is exactly twice your answer to the first part. You may include a graph in your explanation.

2. For the function  $g(x) = \sin(x^2) - 0.5$

A) Plot  $g(x)$  on the domain  $0 \leq x \leq \pi$

B) Compute the definite integral  $\int_0^\pi g(x) dx$

C) Why is the value of the integral negative?

3. It is easy to see that if  $f(x) = 0$ , then  $\int_a^b f(x) dx = 0$ . However, it is not true that having  $\int_a^b g(x) dx = 0$  implies  $g(x)$  is identically zero on the interval  $[a, b]$ . Explain why this is so, and come up with a function and an interval of your own that illustrates this. Include an the integral and plot of your function.

4. For the function

$$f(x) = \begin{cases} -x - 1 & -1 \leq x < 0 \\ x - 1 & 0 \leq x \leq 1 \\ 0 & \textit{otherwise} \end{cases}$$

Compute the integral  $\int_{-2}^2 f(x) dx$  Begin with the following plot command:

```
>plot([[[-2,0], [-1,0]], -x-1, -1+x, [[1,0], [2,0]]], x=-2..2, y=-1..0, color=[red,green,blue,cyan], thickness=5);
```