Solids of Revolution

Introduction

The purpose of this lab is to use Maple to study solids of revolution. Solids of revolution are created by rotating curves in the x-y plane about an axis, generating a three dimensional object.

Two functions about the x-axis

In last week's lab you found the amount removed from a cylindrical piece by subtracting. Looking at this same problem graphed on one axis it will lead to many other similar problems.

```
> with(CalcP7):
> g := x -> 1;j:=x->9/10+sin(2*Pi*x/4)/10;
> revolve({g(x),j(x)},x=0..24,nocap);
```

Click on the graph and drag it around to see it more fully. To solve for the volume you subtract just like last week. Top - Bottom.

Functions about another axis

Another type of Volume of Revolution problem is when the functions are revolved around an axis that is NOT the x-axis. As an example, Find the volme of the function $k(x) = 3x^3 + 16$ for $-1 \leq x \leq 2$ revolved about the horizontal axis $y = 9$.

```
> k := x -> 3*x^3+16;
> plot(k(x),x=-1..2);
> revolve(k(x),x=-1..2,nocap,y=9);
```

The Volume of Revolution formula $\pi \int f(x)^2 dx$ can only be used for revolutions about the x-axis. So move the function!

```
> kshift := x -> k(x)-9;
> revolve(kshift(x),x=-1..2,nocap);
> int(Pi*kshift(x)^2,x=-1..2);
> evalf(%);
```

The volume of $kshift(x)$ is the same as the volume of $k(x)$ which is 1477.

Exercises

1. For the functions $f(x) = 4 - x^2$ and $g(x) = 3$ on the domain $-1 \leq x \leq 1$

    A) Use the `plot` command to graph the two functions. Use the given domain and the range $0 \leq y \leq 4$. Include the option `scaling=constrained` in your plot command.

B) Graph the volume of revolution. (The scaling option doesn't work with this command so right click on the graph and go to scaling-constrained)

C) Find the volume of what looks like a bracelet.

2. For the function $h(x) = cos(\sqrt{x + \pi})(x - 35) - 60$ on the domain $-1 \leq x \leq 100$ Find the Volume of Revolution created when the function is revolved about the horizontal line $y = -20$ by doing the following steps:

A) Use the `plot` command to graph the function on the given domain and the range $-120 \leq y \leq 0$. Include the option `scaling=constrained` in your plot command.

B) Revolve the function about the axis $y = -20$. Right-click for scaling.

C) Create a new function that shifts h(x), then use revolve to show that your new function is the same volume about the x-axis.

D) Find the volume of what looks like a vase.

3. Find the Volume of Revolution created by the functions $a(x) = 2x + 5$ and $b(x) = \frac{x}{3}$ on the domain $-3 \leq x \leq 10$ when rotated about the axis $y = -15$

A) Use the `plot` command to graph the functions on the given domain. Include the option `scaling=constrained` in your plot command.

B) Revolve the functions about the axis $y = -15$. Right-click for scaling.

C) Create new functions that shift $a(x)$ and $b(x)$, then use revolve to show that your new function is the same volume about the x-axis.

D) Find the volume of what looks like a machine part.