Solids of Revolution

Introduction

The purpose of this lab is to use Maple to study solids of revolution. Solids of revolution are created by rotating curves in the x-y plane about an axis, generating a three dimensional object.

Two functions about the x-axis

In last week’s lab you found the amount removed from a cylindrical piece by subtracting. Looking at this same problem graphed on one axis it will lead to many other similar problems.

> with(CalcP7):
> g := x -> 1;j:=x->9/10+sin(2*Pi*x/4)/10;
> revolve({g(x),j(x)},x=0..24,nocap);

Click on the graph and drag it around to see it more fully. To solve for the volume you subtract just like last week. Top - Bottom.

Functions about another axis

Another type of Volume of Revolution problem is when the functions are revolved around an axis that is NOT the x-axis. As an example, Find the volme of the function $k(x) = 3x^3 + 16$ for $-1 \leq x \leq 2$ revolved about the horizontal axis $y = 9$.

> k := x -> 3*x^3+16;
> plot(k(x),x=-1..2);
> revolve(k(x),x=-1..2,nocap,y=9);

The Volume of Revolution formula $\pi \int f(x)^2dx$ can only be used for revolutions about the x-axis. So move the function!

> kshift := x -> k(x)-9;
> revolve(kshift(x),x=-1..2,nocap);
> int(Pi*kshift(x)^2,x=-1..2,nocap);
> evalf(%%);

The volume of $kshift(x)$ is the same as the volume of $k(x)$ which is 1477.

Exercises

1. For the functions $f(x) = 4 - x^2$ and $g(x) = 3$ on the domain $-1 \leq x \leq 1$

   A) Use the plot command to graph the two functions. Use the given domain and the range $0 \leq y \leq 4$. Include the option scaling=constrained in your plot command.
B) Graph the volume of revolution. (The scaling option doesn’t work with this command so right click on the graph and go to scaling-constrained)

C) Find the volume of what looks like a bracelet.

2. For the function \( h(x) = \cos(\sqrt{x + \pi})(x - 35) - 60 \) on the domain \(-1 \leq x \leq 100\)

Find the Volume of Revolution created when the function is revolved about the horizontal line \( y = -20 \) by doing the following steps:

A) Use the \texttt{plot} command to graph the function on the given domain and the range \(-120 \leq y \leq 0\). Include the option \texttt{scaling=constrained} in your plot command.

B) Revolve the function about the axis \( y = -20 \). Right-click for scaling.

C) Create a new function that shifts \( h(x) \), then use revolve to show that your new function is the same volume about the x-axis.

D) Find the volume of what looks like a vase.

3. Find the Volume of Revolution created by the functions \( a(x) = 2x + 5 \) and \( b(x) = \frac{x}{3} \) on the domain \(-3 \leq x \leq 10\) when rotated about the axis \( y = -15 \)

A) Use the \texttt{plot} command to graph the functions on the given domain. Include the option \texttt{scaling=constrained} in your plot command.

B) Revolve the functions about the axis \( y = -15 \). Right-click for scaling.

C) Create new functions that shift \( a(x) \) and \( b(x) \), then use revolve to show that your new function is the same volume about the x-axis.

D) Find the volume of what looks like a machine part.