Assigning labels, expressions and functions

Introduction

The purpose of this lab is to introduce you to the basics needed in any Maple lab. The main topics include: assigning labels, defining expressions, defining functions and plotting.

Entering an expression

Expressions such as $x^3 + 3x^2 - x + 1$ can be entered in a similar way to variable assignment. Choose a variable name to represent the expression and assign the expression to the variable as follows.

> joe := x^3 + 3*x^2 - x + 1;

Entering a function

Suppose you want to enter the same expression stored in $\text{expr}$ above, but as a function of $x$. In Maple, you would type

> f := x -> x^3 + 3*x^2 - x + 1;

Below is how NOT to enter a function:

> f(x) := x^3 + 3*x^2 - x + 1;

The difference between expressions and functions are first the obvious, that expressions do not have to satisfy the definition of a function in the sense that for each input $x$, there is a unique value $f(x)$. A function may be defined as an expression, but not all expressions can be defined as functions. The differences in Maple are numerous as you will see below when we evaluate the expression or function for a given value as well as when using the plot command.

Evaluating functions and expressions

In order to evaluate an expression at a given value of $x$, you must use the subs or eval command. For example, if we wanted to evaluate the expression $\text{expr} = x^3 + 3x^2 - x + 1$ at $x = 2$, the example below show how this can be done

> subs(x=2,joe);
> eval(joe,x=2);
> r := sin(theta) + 8*theta^2;
> subs(theta=1/2*Pi,r);

In the subs command, the first argument tells Maple what you would like to substitute in for $x$. The second argument tells Maple what expression you are substituting into. Note the difference in outputs when a whole number or fraction is entered compared to a decimal.
> \texttt{g:=2*x/3;} \\
> \texttt{subs(x=4,g);} \\
> \texttt{eval(g,x=1/2);} \\
> \texttt{subs(x=4.0,g);} \\

In Maple, functions are much easier to evaluate than expressions. In order to evaluate the function \( f(x) = x^3 + 3x^2 - x + 1 \) at \( x = 2 \), simply type

\[ f(2); \]

Here are a few more examples of evaluating functions.

\[ f(a+h); \]
\[ f(\pi); \]
\[ \texttt{evalf(f(Pi))}; \]

Note the use of the \texttt{evalf} command in the last example above. This command is used when we want Maple to output the answer in \textbf{decimal form}. If this command is not used, the output to your Maple commands will be calculated analytically, where as the \texttt{evalf} command forces Maple to calculate the answers numerically. The \texttt{evalf} command has one essential argument, however a second argument can be added in order to tell how many digits we want to be in the answer. The example below evaluates \( \pi \) using 10 digits, not 10 decimal places.

\[ \texttt{evalf(Pi,10)}; \]

\textbf{Plotting}

The following example shows how the Maple \texttt{plot} command is used to plot the previously entered function \( f(x) \) and expressions \( r \).

\[ \texttt{plot(f(x),x=-5..2)}; \]
\[ \texttt{plot(r,theta=-2*Pi..2*Pi)}; \]

The \texttt{plot} command above has two essential arguments. The first argument is the function(s) or expression(s) that you want to plot. The function or expression can be typed in before the plot command or you can simply type it in as the first argument. The second argument is the range of numbers for the \( x \)-axis. The \texttt{plot} command allows you to add additional optional arguments as well. For instance, you may want to also restrict the \( y \)-axis range or add a title to your plot.

\[ \texttt{plot(x^2,x=-2..2,y=-5..5,title="My First Plot"}); \]

This particular command allows you to add arguments, but if you were to leave off one of the essential arguments, you will get an error message. You can also plot more than one function or expression on the same graph by enclosing them in square brackets [ ] and separating them by commas. For example, we can plot \( f(x) = x^2 - 2 \) and \( g(x) = -x + 2 \) on the same graph.
\begin{verbatim}
> f := x-> x^2-2;
> g := x-> -x+2;
> plot([f(x),g(x)],x=-4..4,color=["DarkOrchid","Gold"]);
> ?plot,colormodes
\end{verbatim}

**Exercises**

1. Enter \( \frac{7x - 3}{2x^2 + 9} \) as an expression.
   
   A) Evaluate this expression at \( x = 0 \).
   
   B) Evaluate this expression at \( x = a + h \).
   
   C) Evaluate this expression at \( x = \frac{1}{4} \).
   
   D) Evaluate this expression at \( x = \sqrt{5} \) and make sure your output is in decimal form.
   
   E) Plot this expression over the \( x \) range \(-10 \leq x \leq 10\).

2. Repeat all parts of exercise 1 by first defining \( \frac{7x - 3}{2x^2 + 9} \) as a function.

3. Define \( x^2 - 6x - 18 \) and \(-2x^2 + 3x + 12\) as functions. Plot them both on the same graph. Estimate the two intersection points by observing the plot. Plug each \( x \) value back into both functions to show that the \( y \) values are the same.