Sequences and Series

Introduction

The purpose of this lab is to learn how to define sequences and series using Maple as well as observe their plots and test for convergence.

Getting Started

To assist you, there is a worksheet associated with this lab that contains examples similar to some of the exercises. On your Maple screen go to File - Open then type the following in the white rectangle:

`\\filer\calclab\MA1023\Seq_Series_start.mw`

You can copy the worksheet now, but you should read through the lab before you load it into Maple. Once you have read through the exercises, start up Maple, load the worksheet, and go through it carefully. Then you can start working on the exercises.

Background

An infinite series is the sum of an infinite sequence. More precisely, the sum of an infinite series is defined as the limit of the sequence of the partial sums of the terms of the series, provided this limit exists. If no finite limit exists, then we say that the series is divergent.

The sum of an infinite series is defined as $\lim_{n \to \infty} S_n$, where S_n is the partial sum of the first n terms of the series. However, because of the algebraic difficulty of expressing S_n as a function of n, it is usually not possible to find sums by directly using the definition.

So, if we can generally not work from the definition, what can be done? There are several convergence tests that provide us with some needed tools. These are tests that tell us if a series converges, but in the case that the series does converge, does not tell us the sum of the series.

The convergence tests that will be used in this lab are the integral test, the ratio test, and the root test.

The Integral Test

The Integral Test for convergence is a method used to test convergence of an infinite series of nonnegative terms.

The series

$$\sum_{n=1}^{\infty} a_n$$

converges if and only if the integral

$$\int_{1}^{\infty} f(x)\, dx$$

is finite, where $f(x)$ is a positive, non-increasing and continuous function defined on the interval $[1, \infty)$ and $f(n) = a_n$ an for all n.

The Ratio Test

The Ratio Test for convergence of a series can be thought of as a measurement of how fast the series is increasing or decreasing. This can be found by looking at the ratio $\frac{a_{n+1}}{a_n}$ as $n \to \infty$.

Given the series $\sum_{n=0}^{\infty} a_n$, suppose that

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = L$$

Then

1. the series converges if $L < 1$,

2. the series diverges if $L > 1$,

3. the test is inconclusive if $L = 1$.

The Root Test

Given the series $\sum_{n=0}^{\infty} a_n$, suppose that

$$\lim_{n \to \infty} \sqrt[n]{|a_n|} = \rho$$

Then

1. the series converges if $\rho < 1$,

2. the series diverges if $\rho > 1$,

3. the test is inconclusive if $\rho = 1$.

Exercises

1. (a) Write the sequence $\frac{7n^2 - 2}{4n^2 + 9}$ as a function.

   (b) Plot the first twenty-five terms.

   (c) Does it appear to converge or diverge? If it seems to converge, to what does it converge?

   (d) Write a function for the sum of the sequence.

   (e) Plot the first twenty-five terms of the sequence of partial sums.

   (f) Does it appear that the series converges or diverges? If it seems to converge, to what does it converge?

2. Apply the integral test to show that the given series converges.

$$\sum_{x=1}^{\infty} \frac{\ln(x+1)^2}{(x+1)^2}$$

   (a) First set up the function.

   (b) Plot the function. Does it seem to satisfy the necesary conditions?

   (c) Take the limit of the integral. What can you concludeand why?

3. Apply the ratio test to the given series.

$$\sum_{x=1}^{\infty} \frac{(2^x)x!}{(x^2)!}$$

   (a) First set up the function.

   (b) Take the limit of the ratio. What do you conclude and why?

4. Apply the root test to the given series.

$$\sum_{x=1}^{\infty} \left(\frac{-n\pi}{\sqrt{9n^2+1}}\right)^{n+1}$$

First set up the function and then take the limit of the root. What do you conclude and why?