Mathematical Basics with Maple

Introduction

The purpose of this lab is to introduce you to the basic commands needed in any Maple lab. There are

- 2 ways to enter a mathematical expression in Maple.
- 3 ways to plug in an x-value to get the y-value.
- 2 ways to find an x-value in an equation algebraically.

Pay close attention in this lab to all the variations in the syntax.

Entering a mathematical expression

Expressions such as $x^3 + 3x^2 - x + 1$ can be entered in a similar way to variable assignment. Choose a variable name to represent the expression and assign the expression to the variable as follows.

> expr := x^3 + 3*x^2 - x + 1;

Entering a function

Suppose you want to enter an expression as a function of x. In Maple you would type the following.

> f := x -> x^2 + x^2 - 6;

The difference between expressions and functions are first the obvious, that expressions do not have to satisfy the definition of a function in the sense that for each input $x$, there is a unique value $y$. A function may be defined as an expression, but not all expressions can be defined as functions. Entering an expression and a function differ as do how they are called up in a Maple command. The function must have the $(x)$ and the expression does not.

> plot([expr, f(x)], x=-6..4);

Evaluating functions and expressions

In order to evaluate an expression at a given x-value, you can use the subs or eval command.

> subs(x=2, expr);
> eval(expr, x=Pi);

In Maple, functions are much easier to evaluate than expressions.
The `evalf` command is used when we want Maple to output the answer in **decimal form**. If this command is not used, the output to your Maple commands will be calculated analytically, where as the `evalf` command forces Maple to calculate the answers numerically. The `evalf` command has one essential argument, however a second argument can be added in order to tell how many digits we want to be in the answer.

```maple
> evalf(eval(expr,x=Pi));
> evalf(eval(expr,x=Pi),20);
```

**Solving a function or an expression algebraically**

You can set an expression or function equal to another expression, function, or number inside a `solve` command. As an example, you may want to find where the following two parabolas intersect.

```maple
> g := 9*x^2-14;
> h:=-x^2;
> plot([g,h],x=-2..2);
> solve(g=h,x);
```

The plot shows that there are two intersection points and the `solve` command finds both \(x\) values. It is good to get into the habit of naming your output so you can use it in a later command. Giving the \(x\) values a name makes it easy to plug them into the function to find the \(y\) values.

```maple
> ip:=solve(g=h,x);
```

Since there are two \(x\) values called `ip`, use `[ ]` to call up the one you want.

```maple
> eval(g,x=ip[1]);
> eval(h,x=ip[2]);
```

Therefore the two intersection points are \((\frac{\sqrt{35}}{5}, \frac{-7}{5})\) and \((-\frac{\sqrt{35}}{5}, \frac{-7}{5})\). This seems like the answer shown on the graph.

**Solving a function or an expression numerically**

If you want to find where the following function crosses the x-axis, just set it equal to zero.

```maple
> f:=theta->-1/2*theta+sin(theta);
> plot(f(theta),theta=-3*Pi..3*Pi);
> solve(f(theta)=0,theta);
```

Wow, what is that?!!? We know from the graph that there should be three answers and `solve` wasn’t a great option so try `fsolve`. 

2
> fsolve(f(theta)=0,theta);

Where are the other two answers!? This is actually how *fsolve* usually works. It shoots for one answer and only gives that one. But you can tell *fsolve* where to look by getting an idea from the graph and typing that domain into the *fsolve* command.

> a:=fsolve(f(theta)=0,theta=-Pi..-1);
> b:=fsolve(f(theta)=0,theta=-1..1);
> c:=fsolve(f(theta)=0,theta=1..Pi);

To find the *y* values just plug in the names of the *x* values.

> f(a);
> f(b);
> f(c);

(Of course the *y*-values are zero!)

### Exercises

1. Given the expression \(4x^6 + \frac{88}{3}x^4 - \frac{172}{3}x^2 - \frac{20}{3}x^5 - \frac{160}{3}x^3 + 60x + 24\),

   A) Plot the expression and in text state how many times the it crosses the *x*-axis.(Experiment with domain values until you find values that show the crossing points clearly.)

   B) Use the Maple *solve* command to find the *x* values of where it crosses the *x*-axis (also called the roots).

   C) Use the Maple *fsolve* command to find the roots.

   D) State, in text, the value of the roots. Also, how are the results of *solve* and *fsolve* different in this problem?

2. Given the functions \(f(x) = \frac{x^4}{7} - 18x \sin(x)\) and \(h(x) = 1.8x - 10\)

   A) Plot the functions. Again experiment with domain values until the intersection points are clear. Then state in text how many intersection points you see.

   B) Using the *solve* command find the intersection points.Label the *x* values by giving the *solve* command a name. How many *x* values does the *solve* command find?

   C) Use the *fsolve* command to find the rest of the answers. Label the *x* values by giving each *fsolve* command a name.

   D) Find all the *y* values and state the intersection points in text.(When writing your text sentence use only two decimal places for the numbers. Round correctly!)
3. Enter the expression $y = 54x^7 + 369x^6 - 4887x^5 - 33597x^4 + 96849x^3 + 760284x^2 + 635040x$

A) Use the factor command to factor the expression. Looking at the output, how many times should the expression cross the x axis?

B) Plot the expression. Again experiment with domain values until the intersection points are clear.

C) Does the solve command find all the roots?