Mathematical Basics with Maple

Introduction

The purpose of this lab is to introduce you to the basic commands needed in any Maple lab. There are

- 2 ways to enter a mathematical expression in Maple.
- 3 ways to plug in an x-value to get the y-value.
- 2 ways to find an x-value in an equation algebraically.

Pay close attention in this lab to all the variations in the syntax.

Entering a a mathematical expression

Expressions such as x^3+3x^2-x+1 can be entered in a similar way to variable assignment. Choose a variable name to represent the expression and assign the expression to the variable as follows.

```
>expr:=x^3+3*x^2-x+1;
```

Entering a function

Suppose you want to enter an expression as a function of x. In Maple you would type the following.

```
>f:=x->x^2+x^2-6;
```

The difference between expressions and functions are first the obvious, that expressions do not have to satisfy the definition of a function in the sense that for each input x, there is a unique value y. A function may be defined as an expression, but not all expressions can be defined as functions. Entering an expression and a function differ as do how they are **called up** in a Maple command. The function must have the (x) and the expression **does not**.

```
>plot([expr,f(x)],x=-6..4);
```

Evaluating functions and expressions

In order to evaluate an expression at a given x-value, you can use the `subs` or `eval` command.

```
>subs(x=2,expr);
>eval(expr,x=Pi);
```

In Maple, functions are much easier to evaluate than expressions.

```
>f(2);
```

The `evalf` command is used when we want Maple to output the answer in **decimal form**. If this command is not used, the output to your Maple commands will be calculated analytically, where as the evalf command forces Maple to calculate the answers numerically. The `evalf` command has one essential argument, however a second argument can be added in order to tell how many digits we want to be in the answer.

```
>evalf(eval(expr,x=Pi));
>evalf(eval(expr,x=Pi),20);
```

Solving a function or an expression algebraically

You can set an expression or function equal to another expression, function, or number inside a `solve` command.As an example, you may want to find where the following two parabolas intersect.

```
> g := 9*x^2-14;
> h:=-x^2;
> plot([g,h],x=-2..2);
> solve(g=h,x);
```

The plot shows that there are two intersection points and the `solve` command finds both x values.It is good to get into the habit of naming your output so you can use it in a later command. Giving the x values a name makes it easy to plug them into the function to find the y values.

```
> ip:=solve(g=h,x);
```

Since there are two x values called ip, use [ ] to call up the one you want.

```
> eval(g,x=ip[1]);
> eval(h,x=ip[2]);
```

Therefore the two intersection points are $(\frac{\sqrt{35}}{5}, \frac{-7}{5})$ and $(\frac{-\sqrt{35}}{5}, \frac{-7}{5})$. This seems like the answer shown on the graph.

Solving a function or an expression numerically

If you want to find where the following function crosses the x-axis, just set it equal to zero.

```
> f:=theta->-1/2*theta+sin(theta);
> plot(f(theta),theta=-3*Pi..3*Pi);
> solve(f(theta)=0,theta);
```

Wow, what is that?!?! We know from the graph that there should be three answers and `solve` wasn't a great option so try `fsolve`.

```
> fsolve(f(theta)=0,theta);
```

Where are the other two answers!? This is actually how `fsolve` usually works. It shoots for one answer and only gives that one. But you can tell `fsolve` where to look by getting an idea from the graph and typing that domain into the `fsolve` command.

```
> a:=fsolve(f(theta)=0,theta=-Pi..-1);
> b:=fsolve(f(theta)=0,theta=-1..1);
> c:=fsolve(f(theta)=0,theta=1..Pi);
```

To find the y values just plug in the names of the x values.

```
> f(a);
> f(b);
> f(c);
```

(Of course the y-values are zero!)

Exercises

1. Given the expression $4x^6 + \frac{88}{3}x^4 - \frac{172}{3}x^2 - \frac{20}{3}x^5 - \frac{160}{3}x^3 + 60x + 24$,

   A) Plot the expression and in text state how many times the it crosses the x-axis.(Experiment with domain values until you find values that show the crossing points clearly.)

   B) Use the Maple `solve` command to find the x values of where it crosses the x-axis (also called the roots).

   C) Use the Maple `fsolve` command to find the roots.

   D) State, in text, the value of the roots. Also, how are the results of `solve` and `fsolve` different in this problem?

2. Given the functions $f(x) = \frac{x^4}{7} - 18xsin(x)$ and $h(x) = 1.8x - 10$

   A) Plot the functions. Again experiment with domain values until the intersection points are clear. Then state in text how many intersection points you see.

   B) Using the `solve` command find the intersection points.Label the x values by giving the `solve` command a name. How many x values does the `solve` command find?

   C) Use the `fsolve` command to find the rest of the answers. Label the x values by giving each `fsolve` command a name.

   D) Find all the y values and state the intersection points in text.(When writing your text sentence use only two decimal places for the numbers. Round correctly!)

3. Enter the expression $y = 54x^7 + 369x^6 - 4887x^5 - 33597x^4 + 96849x^3 + 760284x^2 + 635040x$

   A) Use the factor command to factor the expression. Looking at the output, how many times should the expression cross the x axis?

   B) Plot the expression. Again experiment with domain values until the intersection points are clear.

   C) Does the solve command find all the roots?