Vector Computations

Purpose

The purpose of this lab is to use Maple to introduce you to a number of useful commands for working with vectors. The commands come from the Maple `linalg` package which must be loaded before any of its commands can be used.

Getting Started

To assist you, there is a worksheet associated with this lab that contains examples of the vector commands. On your Maple screen go to File - Open then type the following in the white rectangle:

```
\\filer\calclab\MA1023\Vector_start2.mws My_Documents
```

You can copy the worksheet now, but you should read through the lab before you load it into Maple. Once you have read through the exercises, start up Maple, load the worksheet, and go through it carefully. Then you can start working on the exercises.

Background

The commands below are some of the most basic vector commands. Some examples using these commands can be found in the `Getting Started Worksheet`. More examples can be found in the **Help** screens for each command.

`innerprod`
Computes the dot product (also known as the inner product) of two vectors.
`crossprod`
Computes the cross product of two vectors.
`evalm`
Evaluates expressions involving vectors (and matrices).
`norm`
Computes the norm, or length, of a vector.

Angle between two vectors

If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\theta)$.

Vector Projection

The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ or the component of $\mathbf{b}$ in the direction of $\mathbf{a}$ can be found using the following formula:

$$pr_a\mathbf{b} = (\mathbf{b} \cdot \mathbf{a}/|\mathbf{a}|^2)\mathbf{a}$$

Area of a Triangle

The triangle created by connecting the terminal ends of the vectors **a** and **b** in standard form has area $= \frac{1}{2}|\mathbf{a}X\mathbf{b}|$.

Exercises

1. Given the vectors $\mathbf{a} = [9, 0, -2]$ and $\mathbf{b} = [4, -4, 4]$, and $\mathbf{c} = [\frac{-1}{2}, 3, 0]$, use Maple to compute the value of the following expressions, if possible. For those that cannot be computed because they make no sense, please explain what is wrong.

   A) $12\mathbf{a} - 13\mathbf{b}$

   B) $7\mathbf{c} - 12(\mathbf{a} \cdot \mathbf{c})\mathbf{b}$

   C) $|\mathbf{c}| \cdot \mathbf{a}$

2. For the triangle whose vertices are $\mathbf{Q} = (0, -1, 5)$, $\mathbf{R} = (8, 14, -2)$, and $\mathbf{S} = (-5, -5, -5)$ do the following

   A) Plot the triangle using the following command:

   ```
   >PLOT3D(POLYGONS([Q,R,S]),AXESSTYLE(NORMAL),AXESLABELS(x,y,z),
   VIEW(-10..10,-10..16,-6..10),TEXT(Q, 'Q',COLOR(RGB,1,0,0)),
   TEXT(R, 'R',COLOR(RGB,1,0,0)),TEXT(S, 'S',COLOR(RGB,1,0,0)));
   ```

   B) Find the measure of the three angles in degrees. Use text to keep clear which angle you are working on and label the angle calculation. Remember to shift the vertex to the origin. Finally, Maple uses radians so you will need to convert your calculations to degrees.

   C) Check that your angles add up to 180. Show the computation.

   D) Is the triangle a right triangle, acute triangle, or an obtuse triangle? State your reason.

3. For the parallelogram whose vertices are **p1** = (-3, -15, -1), **p2** = (0.8047704280, -1.597614, 8.7928429), **p3** = (7, 13, 15), and **p4** = (3.1952286, -0.40238569, 5.207157) do the following:

   A) Enter the the vertices as vectors.

   B) Find the area of the parallelogram (Hint:Since a parallelogram is twice a triangle check out the triangle area formula)

   C) A parallelogram is a rhombus if and only if their diagonals are perpendicular. Show whether or not the parallelogram is a rhombus using what you know about vectors. There are two ways of showing this so explain your conclusion.

   D) Graph the parallelogram.

   ```
   >PLOT3D(POLYGONS([p1,p2,p3,p4]),AXESSTYLE(NORMAL),AXESLABELS(x,y,z),
   VIEW(-10..10,-20..20,-1..20),COLOR(ZHUE));
   ```