Polar Coordinates

Background

The use of polar coordinates allows for the analysis of families of curves difficult to handle through rectangular coordinates. If a curve is a rectangular coordinate graph of a function, it cannot have any loops since, for a given x value there can be at most one corresponding y value. However, using polar coordinates, curves with loops can appear as graphs of functions

Plotting Polar Curves

First load the plots package

```
>with(plots):
```

This example is a cardiod.

```
>polarplot(1-cos(theta),theta=0..2*Pi);
```

It is not always apparent what domain you need to get the full curve. You can animate the plot to understand the function better.

```
>animate(polarplot,[1-cos(theta),theta=0..x],x=0..5*Pi/4);
```

To begin the animation, click on the plot and a new tool bar appears where the buttons resemble video controls. Click on the go-arrow. You didn't get the full curve, change the domain until you do.

Area in Polar Coordinates

The relationship between area and integrals in polar coordinates is a little strange; the area inside a circle given (in polar coordinates) by $r = a$ is **NOT** just $\int_0^{2\pi} r d\theta$. Here is the rule: Area inside $r = f(\theta)$ is given by $\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta = \frac{1}{2}\int_{\theta_1}^{\theta_2} f(\theta)^2 d\theta$. This comes from the fact that the area in a thin wedge with radius r and angle $\Delta\Theta$ is $\frac{1}{2}r^2\Delta\Theta$. Note that this gives you the right answer for a circle: $A = \pi r^2$. So to find the area of the cardiod use the following command.

```
>Area1:=1/2*int((1-cos(theta))^2, theta=0..2*Pi);
>evalf(Area1);
```

Exercises

Note: Keep your work well organized and clearly labeled. The equation of a rose is $r = \cos(nt)$. For four consecutive integer values of n do the following four items.

1. Plot the rose using **animate** and find the domain necessary to trace the entire curve without retracing any part of it.

2. For what θ values will begin and end the trace of one petal. (Use the *animate* or use what you know about when cosine equals zero).

3. Plot one petal.

4. Find the area of one petal and then find the area of the entire rose by multiplying or by changing the domain.

A **final question**, how is the value of n related to the number of petals of the rose?