Sequences and Series

Introduction

The purpose of this lab is to learn how to define sequences and series using Maple as well as observe their plots and test for convergence.

Getting Started

To assist you, there is a worksheet associated with this lab that you can copy into your home directory by going to Maple and click on File - Open. In the *File name:* type:

`\\storage\academics\math\calclab\MA1023\Sequence_and_Series_Example_Commands.mw`

when you hit enter the file will load. Click on the gray triangles to read and execute the new commands.

Background

An infinite series is the sum of an infinite sequence. More precisely, the sum of an infinite series is defined as the limit of the sequence of the partial sums of the terms of the series, provided this limit exists. If no finite limit exists, then we say that the series is divergent.

The sum of an infinite series is defined as $\lim_{n\to\infty} S_n$, where S_n is the partial sum of the first n terms of the series. However, because of the algebraic difficulty of expressing S_n as a function of n, it is usually not possible to find sums by directly using the definition.

So, if we can generally not work from the definition, what can be done? There are several convergence tests that provide us with some needed tools. These are tests that tell us if a series converges, but in the case that the series does converge, does not tell us the sum of the series. One of the convergence tests that will be used in this lab is the integral test.

The Integral Test

The Integral Test for convergence is a method used to test convergence of an infinite series of nonnegative terms.

The series

$$\sum_{n=1}^{\infty} a_n$$

converges if and only if the integral

$$\int_1^{\infty} f(x)\,dx$$

is finite, where $f(x)$ is a positive, non-increasing and continuous function defined on the interval $[1,\infty)$ and $f(n) = a_n$ an for all n.

Exercises

1. a) Write the sequence $\dfrac{5\ln(n)+2n}{6n+1}$ as a function.

   b) List the first twenty terms.

   c) Plot the first fifty terms. Does it appear to converge or diverge? If it seems to converge, to what does it converge?

   d) Write a function for the sum of the sequence.

   e) List the first twenty terms of the sequence of partial sums.

   f) Plot the at least the first fifty terms of the sequence of partial sums. Does it appear that the series converges or diverges? If it seems to converge, to what does it converge?

2. For each sereies below, apply the integral test to determine if the given series converges. Be sure to include a plot of $f(x)$ and state whether or not it satisfies necessary conditions for the integral test.

   (a)
   $$\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{(n^3+1)}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{\arctan(n)}{\sqrt{n^2+1}}$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{\sqrt{n}\ln(n)}{(n^2+1)}$$