# L'Hopital's Rule

## Introduction

Maple and L'Hopital's Rule can be used to find the limit of functions in the indeterminate forms of $\frac{0}{0}$ and sometimes $\frac{\infty}{\infty}, \infty 0, \infty - \infty, 0^0$ and $1^\infty$

```
>limit((ln(x-3)/(x^2-16),x=4);
>plot((ln(x-3)/(x^2-16),x=1..6)
```

Looking more closely at the limit we see that the numerator and denominator are both zero thus giving the indeterminate form of $\frac{0}{0}$

```
>limit(ln(x-3),x=4);
>limit(x^2-16,x=4);
```

Applying L'Hopital's Rule, the derivative of the numerator and denominator are taken.

```
>top := diff(ln(x-3), x);
>bottom := diff(x^2-16, x);
>simplify(top/bottom);
```

Now take the limit of the new numerator and denominator and the answer is found.

```
>limit(1, x = 4);
>limit((2*(x-3))*x, x = 4);
```

the value of the limit is $\frac{1}{8}$.

## Exercises

1. For $y = (1+x)^{\frac{1}{x}}$

   A Find the limit at $x = 0$ and then plot the equation on the domain $-2 \leq x \leq 10$ and the range $0 \leq x \leq 40$

   B Find the limit of the base and the exponent individually. Then state the indeterminate form.

   C Taking the natural log of both sides gives you $ln(y) = ln(1+x)^{\frac{1}{x}}$ which can be rewritten as the product (enter it)

   D Take the limit of the numerator and the denominator individually. Then state the indeterminate form.

   E Apply L'Hopital's rule and simplify your new quotient.

   F Take the limit of the numerator and the denominator individually.

   G The natural log was taken to work with the function so it must be undone by using e. Since $\lim_{x\to 0} ln(y) = (youranswer)$ then $\lim_{x\to 0} y = e^{(youranswer)}$. State the answer to the limit.

2. For $y = \frac{ln(1-x)}{cot(\pi x)}$

   A Find the limit at $x = 1$ from the left and then plot the equation on the domain $0 \leq x \leq 1$ and the range $-5 \leq x \leq 5$

   B Take the limit of the numerator and the denominator individually. Then state the indeterminate form.

   C Apply L'Hopital's rule and simplify the new fraction.

   D Take the limit of the numerator and the denominator individually. Then state the indeterminate form.

   E Apply L'Hopital's rule and simplify the new fraction.

   F Take the limit of each factor and state the answer to the limit

3. Create your own indeterminate limit. Show the steps to solve it.