L’Hopital’s Rule

Introduction

Maple and L’Hopital’s Rule can be used to find the limit of functions in the indeterminate forms of \( \frac{0}{0} \) and sometimes \( \frac{\infty}{\infty}, \infty \cdot 0, \infty - \infty, 0^0 \) and \( 1^\infty \)

\[
\text{> limit(((ln(x-3))/(x^2-16)),x=4);} \\
\text{> plot(((ln(x-3))/(x^2-16)),x=1..6)}
\]

Looking more closely at the limit we see that the numerator and denominator are both zero thus giving the indeterminate form of \( \frac{0}{0} \)

\[
\text{> limit(ln(x-3),x=4);} \\
\text{> limit(x^2-16,x=4);} \\
\]

Applying L’Hopital’s Rule, the derivative of the numerator and denominator are taken.

\[
\text{> top := diff(ln(x-3), x);} \\
\text{> bottom := diff(x^2-16, x);} \\
\text{> simplify(top/bottom);} \\
\]

Now take the limit of the new numerator and denominator and the answer is found.

\[
\text{> limit(1, x = 4);} \\
\text{> limit((2*(x-3))*x, x = 4);} \\
\text{the value of the limit is } \frac{1}{8}.
\]

Exercises

1. For \( y = (1 + x)^{\frac{1}{2}} \)
   
   \begin{enumerate}
   \item A Find the limit at \( x = 0 \) and then plot the equation on the domain \(-2 \leq x \leq 10\) and the range \(0 \leq x \leq 40\)
   \item B Find the limit of the base and the exponent individually. Then state the indeterminate form.
   \item C Taking the natural log of both sides gives you \( \ln(y) = \ln(1 + x)^{\frac{1}{2}} \) which can be rewritten as the product (enter it)
   \item D Take the limit of the numerator and the denominator individually. Then state the indeterminate form.
   \item E Apply L’Hopital’s rule and simplify your new quotient.
   \item F Take the limit of the numerator and the denominator individually.
   \item G The natural log was taken to work with the function so it must be undone by using \( e \). Since \( \lim_{x \to 0} \ln(y) = (\text{youranswer}) \) then \( \lim_{x \to 0} y = e^{(\text{youranswer})} \). State the answer to the limit.
   \end{enumerate}
2. For \( y = \frac{\ln(1-x)}{\cot(\pi x)} \)

A Find the limit at \( x = 1 \) from the left and then plot the equation on the domain \( 0 \leq x \leq 1 \) and the range \(-5 \leq x \leq 5\)

B Take the limit of the numerator and the denominator individually. Then state the indeterminate form.

C Apply L’Hopital’s rule and simplify the new fraction.

D Take the limit of the numerator and the denominator individually. Then state the indeterminate form.

E Apply L’Hopital’s rule and simplify the new fraction.

F Take the limit of each factor and state the answer to the limit

3. Create your own indeterminate limit. Show the steps to solve it.