Graphs in Polar Coordinates

Purpose

The purpose of this lab is to help you become familiar with graphs in polar coordinates.

Getting Started

To assist you, there is a worksheet associated with this lab that contains examples and even solutions to some of the exercises. You can copy that worksheet to your home directory with the following command, which must be run in a terminal window, not in Maple.

`\\storage\academics\math\calclab\MA1023\Polar_start.mws`

You can copy the worksheet now, but you should read through the lab before you load it into Maple. Once you have read to the exercises, start up Maple, load the worksheet `Polar_start.mws`, and go through it carefully. Then you can start working on the exercises.

Background

There are many times in math, science, and engineering that coordinate systems other than the familiar one of Cartesian coordinates are convenient. In this lab, we consider one of the most common and useful such systems, that of polar coordinates.

The main reason for using polar coordinates is that they can be used to simply describe regions in the plane that would be very difficult to describe using Cartesian coordinates. For example, graphing the circle $x^2 + y^2 = 1$ in Cartesian coordinates requires two functions - one for the upper half and one for the lower half. In polar coordinates, the same circle has the very simple representation $r = 1$.

Cardioids, Limaçons, and Roses

These are three types of well-known graphs in polar coordinates. The table below will allow you to identify the graphs in the exercises.

Name	Equation
cardioid	$r = a(1 \pm \cos(\theta))$ or $r = a(1 \pm \sin(\theta))$
limaçon	$r = b \pm a\cos(\theta)$ or $r = b \pm a\sin(\theta)$
rose	$r = a\cos(n\theta)$ or $r = a\sin(n\theta)$

Intersections of Curves in Polar Coordinates

Finding where two graphs in Cartesian coordinates intersect is straightforward. You just set the two functions equal and solve for the values of x. In polar coordinates, the situation is more difficult. Most of the difficulties are due to the following considerations.

1. A point in the plane can have more than one representation in polar coordinates. For example, $r = 1$, $\theta = \pi$ is the same point as $r = -1$, $\theta = 0$. In general a point in the plane can have an infinite number of representations in polar coordinates, just by adding multiples of 2π to θ. Even if you restrict θ a point in the plane can have several different representations.

2. The origin is determined by $r = 0$. The angle θ can have any value.

These considerations can make finding the intersections of two graphs in polar coordinates a difficult task. As the exercises demonstrate, it usually requires a combination of plots and solving equations to find all of the intersections.

Exercises

1. For each of the following polar equations, plot the graph in polar coordinates using the `plot` command and identify the graph as a cardioid, limaçon, or rose.

   (a) $r = \cos(2\theta)$

   (b) $r = 1 - 2\sin(\theta)$

   (c) $r = 2 + 2\cos(\theta - \pi/4)$

2. Find all points of intersection for each pair of curves in polar coordinates.

   (a) $r = 1 + \cos(\theta)$ and $r = 3/2$ for $0 \le \theta \le 2\pi$.

   (b) $r = 2 - \cos(\theta)$ and $r = 2\sin(\theta)$ for $0 \le \theta \le \pi$.

   (c) $r = 2 - \sin(\theta)$ and $r = 3\sin(3\theta)$ for $0 \le \theta \le \pi$.

3. Find the domain of θ values that are necessary to trace the inner loop of the limacon $r = 1 - 2\sin(\theta)$. Using these θ values, plot the inner loop in polar coordinates. Then find the area of the inner loop.