Polar Coordinates

Background

The use of polar coordinates allows for the analysis of families of curves difficult to handle through rectangular coordinates. If a curve is a rectangular coordinate graph of a function, it cannot have any loops since, for a given x value there can be at most one corresponding y value. However, using polar coordinates, curves with loops can appear as graphs of functions

Plotting Polar Curves

First load the plots package

```
>with(plots):
```

This example is a cardiod.

```
>polarplot(1-cos(theta),theta=0..2*Pi);
```

It is not always apparent what domain you need to get the full curve. You can animate the plot to understand the function better.

```
>animate(polarplot,[1-cos(theta),theta=0..x],x=0..5*Pi/4);
```

To begin the animation, click on the plot and a new tool bar appears where the buttons resemble video controls. Click on the go-arrow. You didn't get the full curve, change the domain until you do.

Cardioids, Limaçons, and Roses

These are three types of well-known graphs in polar coordinates. The table below will allow you to identify the graphs in the exercises.

Name	Equation
cardioid	$r = a(1 \pm \cos(\theta))$ or $r = a(1 \pm \sin(\theta))$
limaçon	$r = b \pm a\cos(\theta)$ or $r = b \pm a\sin(\theta)$
rose	$r = a\cos(n\theta)$ or $r = a\sin(n\theta)$

Intersections of Curves in Polar Coordinates

Finding where two graphs in Cartesian coordinates intersect is straightforward. You just set the two functions equal and solve for the values of x. In polar coordinates finding intersection points has many considerations.

1. A point in the plane can have more than one representation in polar coordinates. For example, $r = 1$, $\theta = \pi$ is the same point as $r = -1$, $\theta = 0$. In general a point in the plane can have an infinite number of representations in polar coordinates, just by adding multiples of 2π to θ. Even if you restrict θ a point in the plane can have several different representations.

2. The origin is determined by $r = 0$. The angle θ can have any value.

These considerations can make finding the intersections of two graphs in polar coordinates a difficult task. As the exercises demonstrate, it usually requires a combination of plots and solving equations to find all of the intersections.

```
>r1:=t->2-sin(t);r2:=t->2*sin(t);
```

You can use either graphing command to show the intersection points:

```
>polarplot([r1(t),r2(t)],t=0..2*Pi);
>animate(polarplot,[[r1(t),r2(t)],t=0..x],x=0..2*Pi);
```

Use the `fsolve` command so you can set the domain to specify which point you are looking for.

```
>a:=fsolve(r1(t)=r2(t),t=0..2*Pi/2);
>r1(a);r2(a);
```

Substituting into both functions is not necessary.

Area in Polar Coordinates

The relationship between area and integrals in polar coordinates is a little strange; the area inside a circle given (in polar coordinates) by $r = a$ is **NOT** just $\int_0^{2\pi} r d\theta$. Here is the rule: Area inside $r = f(\theta)$ is given by $\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta = \frac{1}{2}\int_{\theta_1}^{\theta_2} f(\theta)^2 d\theta$. This comes from the fact that the area in a thin wedge with radius r and angle $\Delta\Theta$ is $\frac{1}{2}r^2\Delta\Theta$. Note that this gives you the right answer for a circle: $A = \pi r^2$. So to find the area of the above circle use the following command.

```
>1/2*int(r2(t)^2, t=0..Pi);
```

Since the circle is known to be of radius one, we know that the answer is correct.

Exercises

1. For each of the following polar equations, plot the graph in polar coordinates using the `plot` command and identify the graph as a cardioid, limaçon, or rose.

   A) $r = \cos(14\theta)$

   B) $r = 2 - 15\cos(\theta)$

C) $r = 2 + 2\sin(\theta)$

2. Find all points of intersection for each pair of curves in polar coordinates.

   A) $r1 = 1 + \cos(\theta)$ and $r2 = 3/2$ for $0 \leq \theta \leq 2\pi$.

   B) $r3 = 3 - 2\sin(\theta)$ and $r4 = 5\sin(3\theta)$ for $0 \leq \theta \leq 2\pi$.

3. For the function $\mathbf{r4}(t)$ in exercise 2, find the area of one petal of teh rose.