TOPICS

- Business: Course Goals
- A Few Indeterminate Forms
- More Improperness

Business:

- HW Set #1 is due Tuesday ... *at the start of lecture* ...

  Section 9.1: Problems 3, 8, 13, 23

  Section 9.2: Problems 2, 15, 30, 43

- Quiz #1 will be in lecture tomorrow.

  Problems from sections 9.1 and 9.2

  Problems linked to the first course goal

  Closed book, closed notes, no calculator.

- First Computer Lab Today

Some Terminology

- What is a proper integral?
- What is a *improper* integral?
- What is an *indeterminate form?*

  ..... anything you want?

EXAMPLE: From Friday

Evaluate

$$\int_0^\infty xe^{-2x}dx$$

PICTURE:

Area under $xe^{-2x}, \quad x \geq 0$

SOME DETAILS:

Step #1: Truncate the domain of integration:

$$\int_0^\infty xe^{-2x}dx = \lim_{N\to\infty} \int_0^N xe^{-2x}dx$$

MORE DETAILS:

Step #2: Evaluate the (now proper) integral:

$$\begin{aligned}\int_0^N xe^{-2x}dx &= x\frac{-1}{2}e^{-2x}\Big|_0^N - \int_0^N \frac{-1}{2}e^{-2x}dx \\ &= \frac{-N}{2}e^{-2N} + \frac{1}{2}\int_0^N e^{-2x}dx \\ &= \frac{-N}{2}e^{-2N} + \frac{-1}{4}e^{-2x}\Big|_0^N\end{aligned}$$

FINAL DETAILS:

Step #3: Compute a Limit:

$$\begin{aligned}\int_0^\infty xe^{-2x}dx &= \lim_{N\to\infty}\int_0^N xe^{-2x}dx \\ &= \lim_{N\to\infty} -\frac{N}{2}e^{-2N} - \frac{1}{4}e^{-2N} + \frac{1}{4} \\ &=\end{aligned}$$

DETAILS IN THE LIMIT

- $\lim_{N\to\infty} e^{-2N} = 0$

- $\lim_{N\to\infty} Ne^{-2N} = \infty \times 0 = \text{what?}$

What is an INDETERMINATE FORM?

- "$0/0$" = what?
- "∞/∞" = what?
- "$0 \cdot \infty$" = what?
- "0^0" = what?

$$0/0 = x \quad \text{means} \quad x \cdot 0 = 0$$

L'Hopital's Rule

If you have nice functions $f(x)$ and $g(x)$ with

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

then

$$\lim_{x \to a} \frac{g(x)}{f(x)} = \lim_{x \to a} \frac{g'(x)}{f'(x)}$$

if the latter limit exists.

Note: It is still true if you get $\frac{\pm\infty}{\pm\infty}$

A FEW EXAMPLES:

- $\lim_{x \to \infty} xe^{-2x} = ???$

- $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = ???$

Variations on L'Hopital's Rule

Example:

$$\lim_{x \to 0} x^{1/x} =$$

ONE MORE EXAMPLE:

Evaluate the following limit:

$$\lim_{\theta \to 0} \frac{\sin(3\theta)\sqrt{1+\cos(\theta)}}{\theta\sqrt{1+5\theta^4}}$$

BACK TO IMPROPER INTEGRALS

Deal with two problems:

- Infinite limits of integration

- Infinite function values

ONE EXAMPLE:

Compute the following integral:

$$\int_{-1}^{2} x^{-3} dx$$

So Easy!

$$\begin{aligned}
\int_{-1}^{2} x^{-3} dx &= \frac{1}{-2} x^{-2} \Big|_{-1}^{2} \\
&= -\frac{1}{2}(2)^{-2} + \frac{1}{2}(-1)^{-2} \\
&= -\frac{1}{8} + \frac{1}{2} \\
&= \frac{3}{8}
\end{aligned}$$

REMINDERS:

- HW Set #1 is due Tomorrow
- Quiz #1 in lecture tomorrow
- Computer Lab Today