## TOPICS

- Two Kinds of Improper Integrals

  Infinite Limits. . . . . . Infinite Function

- Quiz #1



## Two Pictures:

**OLD EXAMPLE**

**Step #1:** Truncate

$$\int_0^\infty xe^{-2x}dx = \lim_{N\to\infty}\int_0^N xe^{-2x}dx$$

**Step #2:** Evaluate

$$\int_0^N xe^{-2x}dx = -\frac{N}{2}e^{-2N} - \frac{1}{4}e^{-2N} + \frac{1}{4}$$

**Step #3:** Take the limit:

$$\int_0^\infty xe^{-2x}dx = \frac{1}{4}$$



**THE OTHER EXAMPLE:**

Compute the following integral:

$$\int_{-1}^2 x^{-3}dx$$

So Easy!

$$\begin{aligned}\int_{-1}^2 x^{-3}dx &= \left.\frac{1}{-2}x^{-2}\right|_{-1}^{2}\\ &= -\frac{1}{2}(2)^{-2} + \frac{1}{2}(-1)^{-2}\\ &= -\frac{1}{8} + \frac{1}{2}\\ &= \frac{3}{8}\end{aligned}$$

**SO WRONG!**

$$\begin{aligned}
\int_0^2 x^{-3}dx &= \lim_{b\to 0+}\int_b^2 x^{-3}dx \\
&= \lim_{b\to 0+}\left(\frac{1}{-2}x^{-2}\Big|_b^2\right) \\
&= \lim_{b\to 0+}\left(\frac{1}{2}b^{-2}-\frac{1}{8}\right) \\
&=
\end{aligned}$$



**Conclude:**

You just showed that the integral

$$\int_{-1}^2 x^{-3}dx$$

Does Not Exist!

It is actually equal to $\infty - \infty =???$

**NICE EXAMPLE:**

Compute the following integral:

$$\begin{aligned}
\int_0^2 \frac{1}{\sqrt{x}} dx &= \lim_{b \to 0+} \int_b^2 \frac{1}{\sqrt{x}} dx \\
&= \lim_{b \to 0+} \left( 2x^{1/2} \Big|_b^2 \right) \\
&= \lim_{b \to 0+} \left( 2\sqrt{2} - 2b^{1/2} \right) \\
&=
\end{aligned}$$



**QUIZ TIME:**

- Closed Book, Closed Notes, No Calculator
- You have until 9:50pm
- Good Luck!