TOPICS

- Approximating Things
  - $\hookrightarrow$ Functions by Polynomials
  - $\hookrightarrow$ Integrals
  - $\hookrightarrow$ Solutions to Equations
- Taylor Polynomials

Points and Lines and Parabolas

Tangent line is the "best linear approximation" for a function.

To approximate $y = f(x)$ *near* $x = c$, define a line by

$$y - f(c) = f'(c)(x - c)$$

Picture:

Best Line Again...

Find the line that hits the right point with the right slope.

Point: $(c, f(c))$

Slope: $m = f'(c)$

$$\frac{y - f(c)}{x - c} = f'(c)$$

Better Fit...

Find the best quadratic:

$$P_2(x) := a_2x^2 + a_1x + a_0$$

Match value, first derivative, and second derivative:

$$\begin{aligned} f(c) = P_2(c) &= a_2c^2 + a_1c + a_0 \\ f'(c) = P_2'(c) &= 2a_2c + a_1 \\ f''(c) = P_2''(c) &= 2a_2 \end{aligned}$$

EXAMPLE

Approximate $f(x) = \cos(x)$ near $x = c = 0$

$$\begin{aligned} 1 = P_2(0) &= a_0 \\ 0 = P_2'(0) &= a_1 \\ -1 = P_2''(0) &= 2a_2 \end{aligned}$$

Conclude:

$$P_2(x) = 1 + 0x - \frac{1}{2}x^2$$

PICTURES:

Plot $f(x) = \cos(x)$ and $P_2(x) = 1 + 0x - \frac{1}{2}x^2$

KEEP GOING

Why stop at the second derivative?

LOOK FOR THE PATTERN:

Approximate $f(x)$ near $x = 0$

$$\begin{aligned} f(0) = P_2(0) &= a_0 \\ f'(0) = P_2'(0) &= a_1 \\ f''(0) = P_2''(0) &= 2a_2 \\ f'''(0) = P_2'''(0) &= 2 \cdot 3a_3 \end{aligned}$$

THE PATTERN:

$$\begin{aligned} a_0 &= f(0) \\ a_1 &= \frac{f'(0)}{1} \\ a_2 &= \frac{f''(0)}{1 \cdot 2} \\ a_3 &= \frac{f'''(0)}{1 \cdot 2 \cdot 3} \end{aligned}$$

Major Leap:

$$a_n = \frac{f^{(n)}(0)}{n!}$$

SUMMARY

The Taylor Polynomial approximation for $f(x)$ "near" $x = 0$ is given by

$$f(x) \approx a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots a_n x^n$$

where

$$a_n = \frac{f^{(n)}(0)}{n!}$$

EXAMPLE:

For the exponential function:

$$f(x) = e^x \quad \Longrightarrow f^{(n)}(x) = e^x \quad \text{for all} \;\; n$$

So...

$$a_n = \underline{\hspace{6cm}}$$

and so...

REMINDERS

- Homework #2 Due Tomorrow in Conference

  Sec. 9.3: 5, 10, 13;

  Sec. 9.4: 9, 6, 17
- Quiz #2: Tuesday in Lecture
- Make-up Quizzes held in SH 106 from 2:00 to 4:00pm on Friday

  Start (and stop) on the hour and half-hour.