TOPICS

- Solving Equations... Approximately
  - ↪ Bisection Method
  - ↪ Newton's Method
  - ↪ Fixed Point Method
- Quiz #2: Improper Integrals

Picture:

Bisection

1. Look for a sign change—bracket the root;
2. Evaluate at the midpoint—move left or right
3. Do it all again.

Bisection Again

Assume that you have

$$f(0) < 0 \quad \text{and} \quad f(1) > 0$$

Then

- $f(1/2) \_\_\_ 0 \Longrightarrow$ solution is in $[1/2, 1]$
- $f(1/2) \_\_\_ 0 \Longrightarrow$ solution is in $[0, 1/2]$
- Do it again on the smaller interval.

Newton's Method

You are all experts already...

$$f'(x_0) \approx \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

and if you *assume* x_1 gives $f(x_1) = 0$ then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Generalize:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Picture It:

Example:

Build a scheme to approximate the square root of anything.

Idea: If you want $\sqrt{\alpha}$, you need to solve

$$f(x) = x^2 - \alpha = 0$$

Newton Says:

Example:

Fixed Point Methods

Example: You can solve $\cos(\theta) = \theta$ with one finger

1. Put your calculator in radian mode
2. start anywhere (call it θ_0)
3. Hit the cos key until you get tired

Quick Summary:

1. Newton and Bisection will solve $f(x) = 0$
2. Fixed point method will solve $f(x) = x$
3. All methods are recursive (and easy to program).

ANNOUNCEMENTS:

- Homework #3 Due Wednesday in Conference

  Sec. 10.1: 7, 13, 25, 26

  Sec. 10.2: 9, 17, 19

- Quiz #3: Tomorrow in Conference: Taylor Polynomials

- Make-up for Quiz #1: Friday

  2:00, 2:30, 3:00, 3:30

  in Stratton Hall 106