TOPICS

- More Convergence

  $\hookrightarrow$ Sequences of Numbers

- Infinite Series

  $\hookrightarrow$ Geometric First

Example from Lab 2:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots = ???$$

Picture it:

Geometric Series:

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \cdots$$

You can give a simple formula for the sum:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

Derive the Formula:

Truncate:

$$S_N := \sum_{n=0}^{N} r^n = 1 + r + r^2 + r^3 + \cdots r^N$$

Evaluate the Truncated Sum:

$$\begin{array}{rcccccccccccc} S_N & = & 1 & + & r & + & r^2 & + & r^3 & + & \cdots & r^N \\ r \cdot S_N & = & r & + & r^2 & + & r^3 & + & r^4 & + & \cdots & r^{N+1} \end{array}$$

Keep Going:

Compute the Limit:

$$\sum_{n=0}^{\infty} r^n = \lim_{N \to \infty N} S_N = \lim_{N \to \infty N} \frac{1 - r^{N+1}}{1 - r} =???$$

Quick Summary:

1. A Sequence is a LIST of numbers
2. A Series is a SUM of numbers
3. Geometric Series are your friend.

Definition of Convergence

Say that

$$\lim_{n\to\infty} a_n = L$$

if for any number $\epsilon > 0$, there exists a $N > 0$ such that

$$\left|a_n - L\right| < \epsilon \quad \text{whenever} \quad n > N.$$

Apply the Definition:

If $-1 < r < 1$, then

$$\lim_{n\to\infty} r^n = 0$$

Find a formula for N in terms of ϵ.

$$\left|r^N - 0\right| = \left|r\right|^N < \epsilon$$

if and only if

$$N \ln(|r|) < \ln(\epsilon) \iff N > \frac{\ln(\epsilon)}{\ln(|r|)}$$

Summary:

Given $\epsilon > 0$, define N by

$$N > \frac{\ln(\epsilon)}{\ln(|r|)}$$

and you have what you need.

Numbers:

$$r = 0.6 \quad \text{and} \quad \epsilon = 0.001 \Longrightarrow N > \frac{\ln(0.001)}{\ln(0.6)} \approx 2.455$$

Time For Quiz #4:

Any Questions?

ANNOUNCEMENTS:

- Homework #5 Due Friday in Conference

  Section 11.1: 15, 16, 37, 39

  Section 11.2: 23, 24, 25, 41

- Make-up for ALL Quizzes: Friday (Sign up Wednesday)

  2:00, 2:30, 3:00, 3:30

  in Stratton Hall 106