1. Use the integral test to determine whether or not the following series converges. Don’t forget to show that the function determined by the $n$th term of the series satisfies the hypotheses of the integral test.

$$\sum_{k=2}^{\infty} k \exp(-k^2)$$

2. Use the absolute ratio test to determine the interval of convergence of the following power series. You should not examine convergence of the series at the endpoints of the interval of convergence.

(a) $$\sum_{k=0}^{\infty} \frac{k^2}{(2k)!} x^k$$

(b) $$\sum_{k=0}^{\infty} \frac{3^k}{k(k+2)} x^{(2k)}$$

(c) $$\sum_{k=0}^{\infty} \frac{\sqrt{k+1}}{k^2 + 1} x^k$$

3. Determine the order $n$ of the Maclaurin polynomial for $e^x$ that is required to approximate $e^2$ to 3 decimal places. That is, find $n$ so that $|R_n(2)| < 0.0005$.

4. First, write down the Maclaurin series representation for the function $\sin(x)$. Then, use this result to write the Maclaurin series representation for the function $f(x) = x \sin(x^2)$. If you have trouble coming up with a nice formula using summation notation, it is sufficient to write out the first 5 nonzero terms in the series.

5. Use substitution and the Maclaurin series for $1/(1-x)$ to find the first four non-zero terms in the Maclaurin series for $f(x) = \int_{0}^{x} \frac{t}{1+t^3} dt$.

6. Compute the following Taylor polynomials. You may compute them directly, or start from a known series, but make sure you explain your procedure clearly.

(a) Taylor polynomial of order 3 with base point $x = 3$ for the function $f(x) = \frac{2}{x - 1}$

(b) Taylor polynomial of order 4 with base point $x = 0$ for the function $f(x) = \cos(2x)$

7. Consider the following integral.

$$\int_{-2}^{2} \frac{x+1}{x^2 + 2} \, dx$$

Compute approximations to this integral using the trapezoidal rule and Simpson’s rule. Use 4 subintervals in each case.

8. First, write down the general formula for Newton’s method for solving $f(x) = 0$. Then, use a sketch to help explain how the method works. Finally, use two steps of Newton’s method to approximate the root of $g(x) = x^3 - 2x^2 + 4x - 1$ between 0 and 1. Use $x_1 = 1$ as your initial guess.