Parametric curves

\[ x = f(t) \quad t \text{ in } I \]
\[ y = g(t) \quad \text{graph} \]

\[ t = 0 \quad t = 1 \quad t = 2 \]

<table>
<thead>
<tr>
<th>t</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Ex

\[ x = \cos(t) \quad 0 \leq t \leq 2\pi \]
\[ y = \sin(t) \]

if \( t \) is time, parametric curves are about motion.
How about clockwise?

\[ x = \cos(t) \quad 0 \leq t \leq 2\pi \]
\[ y = -\sin(t) \]

\[ t = 0 \]
\[ x(\pi/2) = 0 \]
\[ y(\pi/2) = -1 \]

The curve is always on the circle \( x^2 + y^2 = 1 \). To check, just substitute:

\[(\cos(t))^2 + (\sin(t))^2 = \cos^2(t) + \sin^2(t) = 1\]
Ex
\[ x = t \quad 0 \leq t \leq 2 \]
\[ y = t^2 + 2 \]

Eliminate \( t \):
\[ y = x^2 + 2 \]

Ex
find a parametric description for the circle
\[(x-1)^2 + (y-3)^2 = 4\]
circle of radius 2, center \((1, 3)\)
\[ x = 1 + 2 \cos(t) \quad 0 \leq t \leq 2\pi \\
y = 3 + 2 \sin(t) \]

to show that this is correct, substitute into equation for circle

\[
(2\cos(t))^2 + (2\sin(t))^2 
\]

\[
= 4\cos^2(t) + 4\sin^2(t) 
\]

\[
= 4(\cos^2(t) + \sin^2(t)) 
\]

\[
= 4 
\]

\[
x sext ellipse \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 
\]

\[
x = a \cos(t) \\
y = b \sin(t) \]
Ex

\[ x = \sqrt{1-t^2} \quad -1 \leq t \leq 1 \]

\[ y = t \]

\[ x^2 + y^2 = 1 - t^2 + t^2 = 1 \]

\[ \begin{array}{c}
  t = 1 \\
  t = 0 \\
  t = -1
\end{array} \]

Motion on \( y = h(x) \)

\[ x = f(t) \]

\[ y = h(f(t)) \]
Example motion on $y = x^2 + 1$
go from left to right as $t$ increases.

\[\begin{align*}
x &= t \\
y &= t^2 + 1
\end{align*}\]

motion going from right to left as $t$ increases

\[\begin{align*}
x &= -t \\
y &= (-t)^2 + 1 = t^2 + 1
\end{align*}\]
motion oscillates between \((-1,1)\) and \((1,1)\)

\[
x = \sin(t) \\
y = \sin^2(t) + 1
\]

**Cycloid**

roll the wheel at constant speed

\[\text{at} \]

\[t = 0\]  

\[\text{later time}\]

\[
x = at - a \sin(t) \\
y = a - a \cos(t)
\]
Other example

\[ x = 20 \cos(t) + 5 \cos(8t) \]
\[ y = 20 \sin(t) + 5 \sin(8t) \]