equation for a plane

point $P(x_0, y_0, z_0)$

normal vector $\vec{n} = \langle a, b, c \rangle$

If $Q(x, y, z)$ is in the plane, then we must have

$\vec{n} \cdot (\vec{PQ}) = 0$

so

$\vec{PQ} = \langle x-x_0, y-y_0, z-z_0 \rangle$

$\vec{n} \cdot \vec{PQ} = a(x-x_0) + b(y-y_0) + c(z-z_0)$

so equation of the plane is

$ax + by + cz = ax_0 + by_0 + cz_0$
often written as 
\[ ax + by + cz = d \]

\[ \frac{\text{Ex}}{2x - 3y + 2 = 4} \]
is a plane 
\[ \hat{n} = <2, -3, 17> \]
find a point on the plane 
choose values of \( x, y \), solve for \( z \) 
\[ \text{Ex} \ x = 1, \ y = 1, \ \text{Substitute} \]
\[ 2 \cdot 1 - 3 \cdot 1 + z = 4 \]
or \[ z = 4 - 2 + 3 = 5 \]
point is \( (1, 1, 5) \)
A plane can also be defined by three points

\[ \text{Ex} \]

\[ P(1,2,3), Q(1,4,-1), R(2,7,6) \]

\[ \overrightarrow{PQ} \text{ and } \overrightarrow{PR} \text{ are vectors in the plane, so } \]

\[ \vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} \]

\[ \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -4 \\ 1 & 5 & 3 \end{vmatrix} \]

\[ = (6-(-20))\hat{i} - (0.3-1.(-4))\hat{j} + (0.5-1.2)\hat{k} \]

\[ = 26\hat{i} - 4\hat{j} - 2\hat{k} \]
equation of the plane (using P)

\[ 26(x-1) - 4(y-2) - 2(z-3) = 0 \]

or

\[ 26x - 4y - 2z = 26 - 8 + 6 \]

or

\[ 26x - 4y - 2z = 12 \]

or?

\[ 13x - 2y - z = 6 \]

check: Subs. P, Q, and R
P(1,2,3): 13 - 4 - 3 = 6  ok
vector-valued functions in two dimensions.
\[ \mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} \]
\[ \mathbf{r}(0) = 2\mathbf{i} + 0\mathbf{j} = 2\mathbf{i} \]
\[ \mathbf{r}(\pi/2) = 0\mathbf{i} + 2\mathbf{j} = 2\mathbf{j} \]

graphing: customary to graph coordinates of the head gives parametric curve
\[ x = 2\cos(t), \quad y = 2\sin(t) \]
In three dimensions

\( \vec{r}(t) = 2\cos(t) \hat{i} + 2\sin(t) \hat{j} + t \hat{k} \)

coil spring or helix

In general

\( \vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} + z(t) \hat{k} \)

calculus

\( \vec{r}'(t) = x'(t) \hat{i} + y'(t) \hat{j} + z'(t) \hat{k} \)

\( \frac{\vec{r}'}{\vec{r}(t)} = 2\cos(t) \hat{i} + 2\sin(t) \hat{j} + \hat{k} \)

\( \vec{r}'(t) = -2\sin(t) \hat{i} + 2\cos(t) \hat{j} + \hat{k} \)
in two dimensions

\[ \vec{r}(t) = 2 \cos(t) \hat{i} + 2 \sin(t) \hat{j} \]

\[ \vec{r}'(t) = -2 \sin(t) \hat{i} + 2 \cos(t) \hat{j} \]

\[ \vec{r}'(\pi/2) \text{ at } \pi/2 \]

\[ \vec{r}'(0) \text{ at } t=0 \]

\[ \vec{r}'(0) = 2 \hat{j} \]

\[ \vec{r}'(\pi/2) = -2 \hat{i} \]

\( \vec{r}'(t) \) is a tangent vector

If \( \vec{r}(t) \) is position, \( \vec{r}'(t) \) is velocity, \( \vec{V}(t) \)

\( \vec{r}''(t) \) is \( \vec{V}'(t) \) or acceleration, \( \vec{a}(t) \)
rules for differentiation

1. \( \frac{d}{dt}(\ddot{u}(t) + \ddot{v}(t)) = \ddot{u}'(t) + \ddot{v}'(t) \)

2. \( \frac{d}{dt}(c \ddot{u}) = c \ddot{u}'(t) \)

3. \( \frac{d}{dt}(h(t) \ddot{u}(t)) = h'(t) \ddot{u}(t) + h(t) \ddot{u}'(t) \)

4. \( \frac{d}{dt}(\ddot{u}(t) \cdot \ddot{v}(t)) = \ddot{u}'(t) \cdot \ddot{v}(t) + \ddot{u}(t) \cdot \ddot{v}'(t) \)

5. \( \frac{d}{dt}(\ddot{u}(t) \times \ddot{v}(t)) = \ddot{u}'(t) \times \ddot{v}(t) + \ddot{u}(t) \times \ddot{v}'(t) \)