

MA 1024: Surfaces

Purpose

The purpose of this lab is to introduce you to some of the Maple commands that can be used to plot surfaces in three dimensions.

Background

The graph of a function of a single real variable is a set of points $(x, f(x))$ in the plane. Typically, the graph of such a function is a curve. For functions of two variables in Cartesian coordinates, the graph is a set of points $(x, y, f(x, y))$ in three-dimensional space. For this reason, visualizing functions of two variables is usually more difficult.

One of the most valuable services provided by computer software such as Maple is that it allows us to produce intricate graphs with a minimum of effort on our part. This becomes especially apparent when it comes to functions of two variables, because there are many more computations required to produce one graph, yet Maple performs all these computations with only a little guidance from the user.

The simplest way of describing a surface in Cartesian coordinates is as the graph of a function $z = f(x, y)$ over a domain, e.g. a set of points in the xy plane. The domain can have any shape, but a rectangular one is the easiest to deal with. Another common, but more difficult way of describing a surface is as the graph of an equation $F(x, y, z) = C$, where C is a constant. In this case, we say the surface is defined implicitly. A third way of representing a surface $z = f(x, y)$ is through the use of level curves. The idea is that a plane $z = c$ intersects the surface in a curve. The projection of this curve on the xy plane is called a level curve. A collection of such curves for different values of c is a representation of the surface called a contour plot. Similar to the idea of level curves is to look at cross sections of the surface to see what two-dimensional shape is traced, not only in the xy plane by letting z be constant, but also in the yz plane by holding x constant and the xz plane by holding y constant.

Maple Commands

In order for some of the commands in this lab to work, it will be necessary to load the `plots` package.

```
>with(plots):
```

Some three-dimensional curves can be entered as an explicit function. The plot commands for explicitly defined functions are as follows:

```
>f:=(x,y)->x^2+y^2;  
>plot3d(f(x,y),x=-5..5,y=-5..5);  
>contourplot(f(x,y),x=-5..5,y=-5..5);
```

To look at cross sections of an explicitly defined surface in either of the vertical planes, hold the x or y constant as follows:

```
>plot({f(1,y),f(2,y)},y=-5..5,z=-10..10,labels=[y,z]);
```

Some three-dimensional curves can be entered as an implicit expression, where z is assumed to be a function of x and y . The plot commands for implicitly defined expressions are as follows:

```
>surf:=z^2=x^2+y^2;  
>implicitplot3d(surf,x=-5..5,y=-5..5,z=-5..5,axes=boxed,numpoints=5000);
```

The equation of a sphere can also be entered as an implicit expression.

```
>g:=x^2+y^2+z^2=1;  
>implicitplot3d(g,x=-1..1,y=-1..1,z=-1..1,axes=boxed);
```

To look at the cross-section of the sphere along a plane, hold one variable constant. For example, the intersection of the sphere and the $z = \frac{1}{2}$ plane is:

```
>implicitplot(subs(z=0.5,g),x=-1..1,y=-1..1,labels=[x,y]);
```

Notice that the plot is a two-dimensional circle. To intersect vertical planes for any implicitly defined surface, hold the x or y constant as follows:

```
>implicitplot({subs(y=0.6,g),subs(y=-0.8,g)},x=-1..1,z=-1..1,labels=[x,z]);
```

Other three-dimensional shapes can be made from known conic sections. A few of these will be analyzed in the exercises.

Exercises

1. Generate a surface plot and a contour plot with 30 contours for the following function on the given domain:

$$f(x,y) = \frac{-x}{x^2 + 3y^2 + 4} \quad -5 \leq x \leq 5, \quad -5 \leq y \leq 5$$

- a) Describe the difference in proximity between the contour lines in the regions where the surface plot has a steep incline compared to where the surface plot is almost flat?
- b) What can you say about the surface in the region where the contour plot looks like a series of nested circles?
- c) Rotate the 3-d graph and give an estimate of the extrema. (Extrema are the z values of the highest and lowest points on the graph.)
- d) Visualize the (x,y) coordinate point on the contour plot where you think the extrema occur. Evaluate the function at each of these points and compare to your estimate in part c.

2. For the given equations below, plot each of the following cross sections in two dimension: 2 level curves parallel to the xy plane, 2 level curves xz plane, and 2 level curves parallel to the yz plane. Identify the type or shape of the quadric surface, ie. a sphere, cylinder, cone, elliptic cone, paraboloid, elliptic parabaloid, ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic hyperboloid of one or two sheets, or a hyperbolic parabaloid (saddle). Once you have determined the shape of the surface, supply a three dimensional plot to support your conclusion.

a) Use explicit plotting methods for:

$$z = x^2 + y^2$$

b) Use implicit plotting methods for:

$$z^2 = 1 + x^2 + 4y^2$$