In working the following, use no books or notes. **Show your work.**

1. (10 points) Find \( \lim_{t \to 0} \left[ \left( \frac{t - 1}{t^2 - 1} \right) \mathbf{i} - \frac{\sin t}{t} \mathbf{j} \right] \) or indicate that it does not exist.

   **Solution:**
   
   \[
   \lim_{t \to 0} \left[ \left( \frac{t - 1}{t^2 - 1} \right) \mathbf{i} - \frac{\sin t}{t} \mathbf{j} \right] = \lim_{t \to 0} \left( \frac{t - 1}{t^2 - 1} \right) \mathbf{i} - \lim_{t \to 0} \frac{\sin t}{t} \mathbf{j} = \mathbf{i} - \mathbf{j}.
   \]

   The fact that \( \lim_{t \to 0} \frac{\sin t}{t} = 1 \) can be verified with L'Hôpital's Rule (and was on several homework problems).

2. (10 points) Use the given information to find the velocity vector \( \mathbf{v}(t) \) and the position vector \( \mathbf{r}(t) \):

   \[
   \mathbf{a}(t) = \mathbf{i} + e^{-t} \mathbf{j}, \quad \mathbf{v}(0) = 2\mathbf{i} + \mathbf{j}, \quad \mathbf{r}(0) = \mathbf{i} + \mathbf{j}
   \]

   **Solution:** We have \( \mathbf{v}(t) = \int \mathbf{a}(u) \, du = t\mathbf{i} - e^{-t} \mathbf{j} + \mathbf{C} \). Then \( \mathbf{v}(0) = 0 \cdot \mathbf{i} - \mathbf{j} + \mathbf{C} = 2\mathbf{i} + \mathbf{j} \), so \( \mathbf{C} = 2\mathbf{i} + 2\mathbf{j} \) and

   \[
   \mathbf{v}(t) = (2 + t)\mathbf{i} + (2 - e^{-t})\mathbf{j}.
   \]

   Continuing, we have \( \mathbf{r}(t) = \int \mathbf{v}(u) \, du = (2t + \frac{1}{2}t^2)\mathbf{i} + (2t + e^{-t})\mathbf{j} + \mathbf{K} \). Then \( \mathbf{r}(0) = 0 \cdot \mathbf{i} + \mathbf{j} + \mathbf{K} = \mathbf{i} + \mathbf{j} \), so \( \mathbf{K} = \mathbf{i} \) and

   \[
   \mathbf{r}(t) = (1 + 2t + \frac{1}{2}t^2)\mathbf{i} + (2t + e^{-t})\mathbf{j}.
   \]