MA 1024            Quiz 3            November 8, 2002

In working the following, use no books, notes, or calculators. **Show your work.**

Note: As in the text, boldface is used to denote vectors.

1. (2 points each) Find the following for \( \mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k} \). DON'T PANIC.
   (a) the velocity \( \mathbf{v}(t) = \mathbf{r}'(t) \),
   \( \text{Solution:} \quad \mathbf{v}(t) = -\sin t \mathbf{i} + \cos t \mathbf{j} + \mathbf{k}. \)
   (b) the speed \( \frac{ds}{dt} = |\mathbf{v}(t)| = |\mathbf{r}'(t)|, \)
   \( \text{Solution:} \quad \frac{ds}{dt} = |\mathbf{v}(t)| = \sqrt{\sin^2 t + \cos^2 t + 1^2} = \sqrt{2}. \)
   (c) the unit tangent \( \mathbf{T}(t) \),
   \( \text{Solution:} \quad \mathbf{T}(t) = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|} = \frac{-\sin t}{\sqrt{2}} \mathbf{i} + \frac{\cos t}{\sqrt{2}} \mathbf{j} + \frac{1}{\sqrt{2}} \mathbf{k}. \)
   (d) the acceleration \( \mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) \),
   \( \text{Solution:} \quad \mathbf{a}(t) = -\cos t \mathbf{i} - \sin t \mathbf{j}. \)
   (e) the tangential and normal components of acceleration \( a_T \) and \( a_N \).
   \( \text{Solution: From the formula sheet,} \)
   \[
   a_T = \mathbf{T} \cdot \mathbf{a} = \left( \frac{-\sin t}{\sqrt{2}} \mathbf{i} + \frac{\cos t}{\sqrt{2}} \mathbf{j} + \frac{1}{\sqrt{2}} \mathbf{k} \right) \cdot \left( -\cos t \mathbf{i} - \sin t \mathbf{j} \right) = \frac{\sin t \cos t}{\sqrt{2}} - \frac{\cos t \sin t}{\sqrt{2}} + 0 = 0
   \]
   and \( a_N = \sqrt{|a|^2 - a_T^2} = \sqrt{\cos^2 t + \sin^2 t} = 1. \)

2. (10 points) Find the equation of the surface that results when the curve \( 4z^2 + 3y^2 = 12 \) is revolved about the y-axis.
   \( \text{Solution: The equation of the surface is} \quad 4(z^2 + z^2) + 3y^2 = 12, \) or \( 4z^2 + 3y^2 + 4z^2 = 12. \)