MA 1024: Surfaces

Purpose

The purpose of this lab is to introduce you to some of the Maple commands that can be used to plot surfaces in three dimensions.

Background

For functions of two variables in Cartesian coordinates, the graph is a set of points $(x, y, f(x, y))$ in three-dimensional space. For this reason, visualizing functions of two variables is usually more difficult.

One of the most valuable services provided by computer software such as Maple is that it allows us to produce intricate graphs with a minimum of effort on our part. This becomes especially apparent when it comes to functions of two variables, because there are many more computations required to produce one graph, yet Maple performs all these computations with only a little guidance from the user.

The simplest way of describing a surface in Cartesian coordinates is as the graph of a function $z = f(x, y)$ over a domain, e.g. a set of points in the xy plane. The domain can have any shape, but a rectangular one is the easiest to deal with. Another common, but more difficult way of describing a surface is as the graph of an equation $F(x, y, z) = C$, where C is a constant. In this case, we say the surface is defined implicitly. A third way of representing a surface $z = f(x, y)$ is through the use of level curves. The idea is that a plane $z = c$ intersects the surface in a curve. The projection of this curve on the xy plane is called a level curve. A collection of such curves for different values of c is a representation of the surface called a contour plot. Similar to the idea of level curves is to look at cross sections of the surface to see what two-dimensional shape is traced, not only in the xy plane by letting z be constant, but also in the yz plane by holding x constant and the xz plane by holding y constant.

Defining Functions of Two Variables

Defining functions of two variables in Maple is very similar to the way it is done for functions of one variable - just remember the parentheses.

```
>f:=(x,y)->x^2+y^2;
```

Evaluating the function at a specific (x,y) value is easy:

```
>f(3,1);
```

Plotting Three-dimensional Surfaces

The **plot3d** command is similar to the **plot** command except the domain has both x and y values.However, there is a lot more you can do with plot3d. First try moving the plot by clicking and dragging. Also, a menu of options will appear if you right click on the graph.

```
>plot3d(f(x,y),x=-2..2,y=-2..2);
```

Instead of using the right-click menu you can put the options into the plot command.

```
plot3d(f(x,y),x=-2..2,y=-2..2,scaling=constrained,axes=boxed,color=magenta);
```

Cross Sections

The easiest way to get cross sections (parallel to the x-y plane) is to use the Maple command **contourplot** which is included in the package **plots**. The following command will show 15 cross sections using z-values that the computer will choose.

```
>with(plots):
>contourplot(f(x,y),x=-2..2,y=-2..2,contours=15);
```

Thefollowing command will show 3 cross sections using z-values that you choose.

```
>contourplot(f(x,y),x=-2..2,y=-2..2,contours=[1,0.8,3]);
```

In the above commands the z-value was held constant thus giving a two-dimensional plot. You can hold x or y constant to get a cross section perpendicular to the y-z or x-z plane. To get these cross sections use the **plot** command.

```
>plot(f(5,y),y=-2..2,labels=[y,z]);
```

Note how easy it is to hold the x (or y) constant when you have entered a function, **f(5,y)**. Also note that the axes were labeled to emphasize what the two remaining variables are. Next is an example of four contours parallel to the x-z plane.

```
>plot({f(x,0.8),f(x,0),f(x,1),f(x,2)},x=-2..2,labels=[x,z]);
```

Exercises

1. Generate a surface plot and contour plot with 22 contours for the following function on the given domain:

$$f(x,y) = (x^2 - y^2)e^{(-x^2-y^2)}, \quad -3 \leq x \leq 3, \quad -3 \leq y \leq 3$$

   A Describe the difference between the contour lines in the regions where the surface plot has a steep incline compared to where the surface plot is almost flat?

   B What must exist inside the region where the contour plot looks like a series of nested circles?

2. For the function

$$g(x,y) = -\frac{1}{3}x^2 - \frac{1}{5}y^2 + 5, \quad -5 \leq x \leq 5, \quad -5 \leq y \leq 5$$

   ,(use the option **,scaling=constrained** in all plot commands)

A Plot 2 two-dimensional level curves parallel to the xy plane and state what shapes are graphed.

B Plot 2 two-dimensional cross sections parallel to the xz plane and state what shapes are graphed.

C Plot 2 two-dimensional cross sectionas parallel to the yz plane and state the shapes graphed.

D Identify the type or shape of the quadric surface, ie. a sphere, cylinder, cone, elliptic cone, paraboloid, elliptic parabaloid, ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic hyperboloid of one or two sheets, or a hyperbolic parabaloid (saddle). Once you have determined the shape of the surface, supply a three dimensional plot to support your conclusion.

3. Create a contour plot for the function $h(x, y) = \dfrac{x - y}{x^2 + y^2}$ for the z values -2, -1, -1/2, 1/2, 1, 2 using Maple's `contourplot` command. Choose a domain that will show the contours clearly. Use the option **,numpoints=2000** in your **contourplot** command.