Polar Coordinates

Background

The use of polar coordinates allows for the analysis of families of curves difficult to handle through rectangular coordinates. If a curve is a rectangular coordinate graph of a function, it cannot have any loops since, for a given x value there can be at most one corresponding y value. However, using polar coordinates, curves with loops can appear as graphs of functions

Plotting Polar Curves

When you graph curves in polar coordinates, you are really working with parametric curves. The basic idea is that you want to plot a set of points by giving their coordinates in (x, y) pairs. When you use polar coordinates, you are defining the points (x, y) in terms of polar coordinates (r, θ). When you plot polar curves, you are usually assuming that r is a function of the angle θ and θ is the parameter that describes the curve. In Maple you have to put square brackets around the curve and add the specification *coords=polar*. Maple assumes that the first coordinate in the parametric plot is the radius (r) and the second coordinates is the angle (θ).

Cardioids, Limaçons, and Roses

These are three types of well-known graphs in polar coordinates. The table below will allow you to identify the graphs in the exercises.

Name	Equation
cardioid	$r = a(1 \pm \cos(\theta))$ or $r = a(1 \pm \sin(\theta))$
limaçon	$r = b \pm a\cos(\theta)$ or $r = b \pm a\sin(\theta)$
rose	$r = a\cos(n\theta)$ or $r = a\sin(n\theta)$

Below is an example of a cardiod.

```
>with(plots):
>polarplot(1-cos(theta),theta=0..2*Pi);
```

Area in Polar Coordinates

The relationship between area and integrals in polar coordinates is a little strange; the area inside a circle given (in polar coordinates) by $r = a$ is **NOT** just $\int_0^{2\pi} r\,d\theta$. Here is the rule: Area inside $r = f(\theta)$ is given by $\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta = \frac{1}{2}\int_{\theta_1}^{\theta_2} f(\theta)^2 d\theta$. This comes from the fact that the area in a thin wedge with radius r and angle $\Delta\Theta$ is $\frac{1}{2}r^2\Delta\Theta$. Note that this gives you the right answer for a circle: $A = \pi r^2$. So to find the area of the cardiod use the following command.

```
>1/2*int((1-cos(theta))^2, theta=0..2*Pi);
```

When doing double integration the difference of the outer and inner radius changes the formula to $\int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} r dr d\theta$

```
>int(int(r,r=0..1-cos(theta)),theta=0..2*Pi);
```

Exercises

1. For each of the following polar equations, plot the graph in polar coordinates using the `polarplot` command and identify the graph as a cardioid, limaçon, or rose.

   (a) $r = \cos(2\theta)$

   (b) $r = 2 - 3\cos(\theta)$

   (c) $r = 1 + \sin(\theta + \pi/4)$

   (d) $r = 3\sin(4\theta)$

2. Find all points of intersection for each pair of curves in polar coordinates.You will need to use the display command to plot both functions on the same graph.

   (a) $r = 1 + \cos(\theta)$ and $r = 3/2$ for $0 \leq \theta \leq 2\pi$.

   (b) $r = 1 - \sin(\theta)$ and $r = 3/2 + \cos(\theta)$ for $0 \leq \theta \leq 2\pi$.

   (Your answers should be given in proper polar form (r, θ).)

3. Find the angles that create only one petal of the five petal rose given by the equation $r = 2\cos(5\theta)$. Plot only one petal and find the area of that petal using single integration and then double integration.