Double integrals with Maple

Purpose

The purpose of this lab is to acquaint you with using Maple to do double integrals.

Maple Commands

The main command for computing multiple integrals with Maple is the **int** command you are already familiar with. You simply use nested **int** commands and compute the integrals as iterated integrals. To integrate a function over a rectangular region, just nest the two **int** commands. The following command computes the integral $\int_0^1 \int_{-2}^1 x^2+y^2\,dx\,dy$

```
>int(int(x^2+y^2,x=-2..1),y=0..1);
```

This command computes the same integral, but in the opposite order. that is, it computes $\int_{-2}^1 \int_0^1 x^2+y^2\,dy\,dx$

```
>int(int(x^2+y^2,y=0..1),x=-2..1);
```

Maple can also compute double intgrals where the limits are not constants. For example, suppose you wanted to compute the integral of $f(x,y) = x^2+y^2$ over the disk in the x-y plane whose boundary is the circle $(x-1)^2+y^2=1$. This can be treated as a region that is y-simple by solving the equation of the circle for y. This gives two functions $y = \sqrt{1-(x-1)^2}$ and $y = -\sqrt{1-(x-1)^2}$, which are just the upper and lower halves of the circle. The integral would be $\int_0^2 \int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}} x^2+y^2\,dy\,dx$ and the Maple command to do this integral is

```
>int(int(x^2+y^2,y=-sqrt(1-(x-1)^2)..sqrt(1-(x-1)^2)),x=0..2);
```

You can also use Maple to compute double integrals over regions that are x-simple. Suppose we repeat the previous calculation, but solve the equation for x instead of y. This gives the two functions $x = 1+\sqrt{1-y^2}$ and $x = 1-\sqrt{1-y^2}$ for the right and left halves of the circle. The integral would be $\int_{-1}^1 \int_{1-\sqrt{1-y^2}}^{1+\sqrt{1-y^2}} x^2+y^2\,dx\,dy$ and the Maple command to do this is:

```
>int(int(x^2+y^2,x=1-sqrt(1-y^2)..1+sqrt(1-y^2)),y=-1..1);
```

Exercises

1. Let R be the region in the xy plane bounded by the two curves $y=\sqrt{4-x^2}$ and $y = -\frac{1}{3}x+2$. Use a double integral to compute the area of the region.
2. Using double integration find the area of the triangle bounded by $2x+3y=6$, $x=-2$, and $y=0$. Compute the integral using y as the inner variable of integration and then repeat the calculuation using x as the inner variable of integration. You should get the same answer.

3. Use a double integral to find the volume of the region bounded by the two paraboloids $z = x^2 + 2y^2$ and $z = 12 - 2x^2 - y^2$.