MA 1024: Surfaces

Purpose

The purpose of this lab is to introduce you to some of the Maple commands that can be used to plot surfaces in three dimensions.

Background

To graph in three dimensions there are two important commands: **plot3d** and **implicitplot3d**. Check out the **plot3d**:

```
>with(plots):
>f:=(x,y)->x^2+3.7*y^2;
plot3d(f(x,y),x=-5..5,y=-3..3,style=patchnogrid);
```

The **implicitplot3d** has the important difference of using an entire equation. In other words the equal sign MUST be used. You can use an expression to enter the entire equation.

```
>blob:=z=x^2+3.7*y^2;
>implicitplot3d(blob,x=-5..5,y=-3..3,z=0..60);
```

Sometimes one command is better than the other. For example, to graph a sphere, using two functions can be cumbersome and may just look bad. In this case the **implicitplot3d** is much better.

```
>g1:=(x,y)->sqrt(1-x^2-y^2);g2:=(x,y)->-sqrt(1-x^2-y^2);
>plot3d([g1(x,y),g2(x,y)],x=-1..1,y=-1..1);
>g3:=1=x^2+y^2+z^2;
>implicitplot3d(g3,x=-1..1,y=-1..1,z=-1..1,style=surface,numpoints=5000);
```

Another example: The **implicitplot3d** can graph functions as long as you finish the equation in the graphing command. Notice the equals z.

```
>h:=(x,y)->7*x+y;
>implicitplot3d([g3,h(x,y)=z],x=-1.2..1.2,y=-1.2..1.2,z=-1.2..1.2,
style=[wireframe,surface],color=[magenta,blue],numpoints=3000);
```

Cross Sections

To graph the two-dimensional cross sections, once again the commands differ between a function and an implicit equation. For functions use the **contourplot** and **plot** commands.

```
>contourplot(f(x,y),x=-3..3,y=-3..3,contours=[3,6],scaling=constrained);
```

NOTE! **contourplot** will ONLY work with functions and ONLY give the z cross sections. To get the x or y cross sections use **plot**.

```
>plot([f(-2,y),f(0,y),f(3,y)],y=-3..3);
```

For implicit equations use the **implicitplot** command for the x, y, and z cross sections.

```
>implicitplot(subs(z=0.5,g3),x=-1..1,y=-1..1,labels=[x,y]);
>implicitplot({subs(y=0.6,g3),subs(y=-0.8,g3)},x=-1..1,z=-1..1,labels=[x,z]);
```

Exercises

1. Given $z^2 = 70 - 3.25x^2 - 7.4y^2$

   A) Using **implicitplot3d** and all the appropriate related commands, graph the three-dimensional shape over the intervals $-5 \leq x \leq 5$, $-4 \leq y \leq 4$, and $-10 \leq z \leq 10$.

   B) Is the given equation a function?

   C) Plot the intersections of this shape and two planes perpendicular to the z-axis. What two-dimensional shapes are graphed?

   D) Plot the intersections of this shape and two planes perpendicular to the y-axis. What two-dimensional shapes are graphed?

   E) Plot the intersections of this shape and two planes perpendicular to the x-axis. What two-dimensional shapes are graphed?

   F) What three-dimensional shape is the equation (a sphere, a paraboloid, an elliptic paraboloid, an ellipsoid, an hyperboloid, an elliptic hyperboloid, or an hyperbolic paraboloid (saddle))?

2. Given $z = \frac{x^2}{4} - \frac{y^2}{20}$

   A) Using **plot3d** and all the appropriate related commands, graph the three-dimensional shape over the intervals $-10 \leq x \leq 10$, and $-10 \leq y \leq 10$.

   B) Is the given equation a function?

   C) Plot the intersections of this shape and two planes perpendicular to the z-axis. What two-dimensional shapes are graphed?

   D) Plot the intersections of this shape and two planes perpendicular to the y-axis. What two-dimensional shapes are graphed?

   E) Plot the intersections of this shape and two planes perpendicular to the x-axis. What two-dimensional shapes are graphed?

   F) What three-dimensional shape is the equation (a sphere, a paraboloid, an elliptic paraboloid, an ellipsoid, an hyperboloid, an elliptic hyperboloid, or an hyperbolic paraboloid (saddle))?

3. Given $-y^2 + x^2 = 1.7z^2 + 3.7$

   A) Plot the three-dimensional shape over the intervals $-10 \leq x \leq 10$, $-10 \leq y \leq 10$, and $-5 \leq z \leq 10$.

B) Is the given equation a function?

C) Plot the intersections of this shape and two planes perpendicular to the z-axis. What two-dimensional shapes are graphed?

D) Plot the intersections of this shape and two planes perpendicular to the y-axis. What two-dimensional shapes are graphed?

E) Plot the intersections of this shape and two planes perpendicular to the x-axis. What two-dimensional shapes are graphed?

F) What three-dimensional shape is the equation (a sphere, a paraboloid, an elliptic paraboloid, an ellipsoid, an hyperboloid, an elliptic hyperboloid, or an hyperbolic paraboloid (saddle))?