Cylindrical and Spherical Coordinates

Background

Defining surfaces with rectangular coordinates often times becomes more complicated than necessary. A change in coordinates can simplify things. The easiest examples are a sphere and a cylinder.

```
> with(plots):
> f1:=x^2+y^2+z^2=49;
> g1:=rho=7;
> implicitplot3d(f1,x=-7..7,y=-7..7,z=-7..7,axes=boxed,scaling=constrained);
> implicitplot3d(g1,rho=0..7.5,theta=0..2*Pi,phi=0..Pi,coords=spherical,
numpoints=5000,axes=boxed);
> f2:=x^2+y^2=49;
> h2:=r=7;
> implicitplot3d(f2,x=-7..7,y=-7..7,z=-8..8,axes=boxed);
> implicitplot3d(h2,r=0..7.5,theta=0..2*Pi,z=-8..8,coords=cylindrical,
numpoints=3000,axes=boxed);
```

To change to cylindrical coordinates from rectangular coordinates use the conversion:

$$\boxed{\begin{array}{c} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = z \end{array}}$$

Where r is the radius in the x-y plane and θ is the angle in the x-y plane. To change to spherical coordinates from rectangular coordinates use the conversion:

$$\boxed{\begin{array}{c} x = \rho\sin(\phi)\cos(\theta) \\ y = \rho\sin(\phi)\sin(\theta) \\ z = \rho\cos(\phi) \end{array}}$$

Where θ is the angle in the x-y plane; ρ is the radius from the origin in any direction; and ϕ is the angle in the x-z plane. As an example, the equation of an ellipsoid in rectangular coordinates is

$$\frac{x^2}{23} + \frac{y^2}{23} + \frac{z^2}{122} = 1$$

```
> f3:=x^2/23+y^2/23+z^2/122=1;
>implicitplot3d(f3,x=-5..5,y=-5..5,z=-12..12,scaling=constrained,axes=boxed);
```

Changing to sherical coordinates:

```
> g3:=simplify(subs({x=rho*sin(phi)*cos(theta),y=rho*sin(phi)*sin(theta),
z=rho*cos(phi)},f3));
> implicitplot3d(g3,rho=0..12,theta=0..2*Pi,phi=0..Pi,coords=spherical,axes=boxed,
scaling=constrained,numpoints=2000);
```

Exercises

1. Given the rectangular equation:

$$e^{-x^2-y^2} = z$$

   A) Graph the equation using the domain values of $-2 \leq x \leq 2$, $-2 \leq y \leq 2$ and the range values $0 \leq z \leq 1$.

   B) Write the equation in spherical coordinates and then graph the equation.

   C) Write the equation in cylindrical coordinates and graph it.

   D) Looking at the three equations, which coordinates appears to give the simplest equation?

2. Given the equation:

$$14(3x)^2 + 7(4y^2) = (5x^2 + z^2 + y^4)^2$$

   A) Graph the equation using the domain values $-2.5 \leq x \leq 2.5$, $-2 \leq y \leq 2$ and the range values $-3 \leq z \leq 3$.

   B) Write the equation in spherical coordinates and graph it.

   C) Write the equation in cylindrical coordinates. Then graph your equation.

   D) Looking at the three equations, which coordinates appear to give the simplest equation?

3. Given the equation:

$$\sqrt{x^2 + y^2 + z^2} = 10 + 2\cos(4\arctan(\frac{y}{x}))\sin(4\arccos(\frac{z}{\sqrt{x^2 + y^2 + z^2}}))$$

   A) Graph the equation using the domain values $-15 \leq x \leq 15$, $-15 \leq y \leq 15$ and the range values $-15 \leq z \leq 15$.

   B) Write the equation in spherical coordinates and graph it (note: the csgn is simply a sign note to the computer).

   C) Write the equation in cylindrical coordinates. Then graph your equation.

   D) Looking at the three equations, which coordinates appear to give the simplest equation? (Hint: are there some easy trig simplifications you can make?)