Solving DE by SOV and modeling with DE.

2/7/08

Ex

$$y' = ky, \quad k > 0$$

To solve by SOV, write

$$\frac{dy}{dt} = ky$$

rewrite as

$$\frac{1}{y}\,dy = k\,dt$$

now, integrate both sides

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln|y| = kt + c_1$$

so

$$|y| = e^{c_1} e^{kt}$$

Don't want $|\cdot|$, note that $e^{c_1} > 0$, so we replace e^{c_1} by c, and allow c to be pos. or negative, This lets us drop the $|\cdot|$, giving

$$y = c e^{kt}$$

The value of c is determined by an I.C.

<u>Ex</u>

solve

$$\frac{dy}{dt} = \frac{1}{2}y, \quad y(1) = 2$$

$k = \frac{1}{2}$ so

$$y = Ce^{\frac{1}{2}t}$$

Apply I.C.

$$y(1) = Ce^{\frac{1}{2}} = 2$$

so $C = 2e^{-\frac{1}{2}}$

and our solution is

$$y = 2e^{-\frac{1}{2}}e^{\frac{1}{2}t} = 2e^{\frac{1}{2}(t-1)}$$

modeling:

The rate of change of $P(t)$, the number of bacteria in a culture medium is proportional to $P(t)$

write a DE for this situation.

$$\left\{\begin{matrix}\text{rate of} \\ \text{change} \\ \text{of } P\end{matrix}\right\} = k\{P\}$$

or

$$\frac{dP}{dt} = kP$$

Glucose in the bloodstream

Glucose is infused at a const. rate C g/min and the amount, $A(t)$, of glucose in the bloodstream is converted and removed at a rate prop. to the amount present.

$$\left\{\begin{matrix}\text{rate of}\\ \text{change}\\ \text{of } A\end{matrix}\right\} = \left\{\begin{matrix}\text{rate}\\ \text{of}\\ \text{addition}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate}\\ \text{of}\\ \text{conv.}\end{matrix}\right\}$$

as a DE, get

$$\frac{dA}{dt} = C - kA$$

To solve, separate var.

$$\frac{dA}{C-KA} = dt$$

integrate, get

$$-\frac{1}{K}\ln|C-KA| = t + c_1$$

or

$$\ln|C-KA| = -Kt + c_2$$

so

$$|C-KA| = e^{c_2}e^{-Kt}$$

if we allow c_3 to be + or −, can write

$$C-KA = c_3 e^{-Kt}$$

so

$$C - c_3 e^{-kt} = kA$$

or

$$A = \frac{C}{k} - \frac{c_3}{k} e^{-kt}$$

I.C. $A(0) = A_0$, have

$$A_0 = \frac{C}{k} - \frac{c_3}{k} \cdot 1$$

so

$$\frac{c_3}{k} = \frac{C}{k} - A_0$$

or

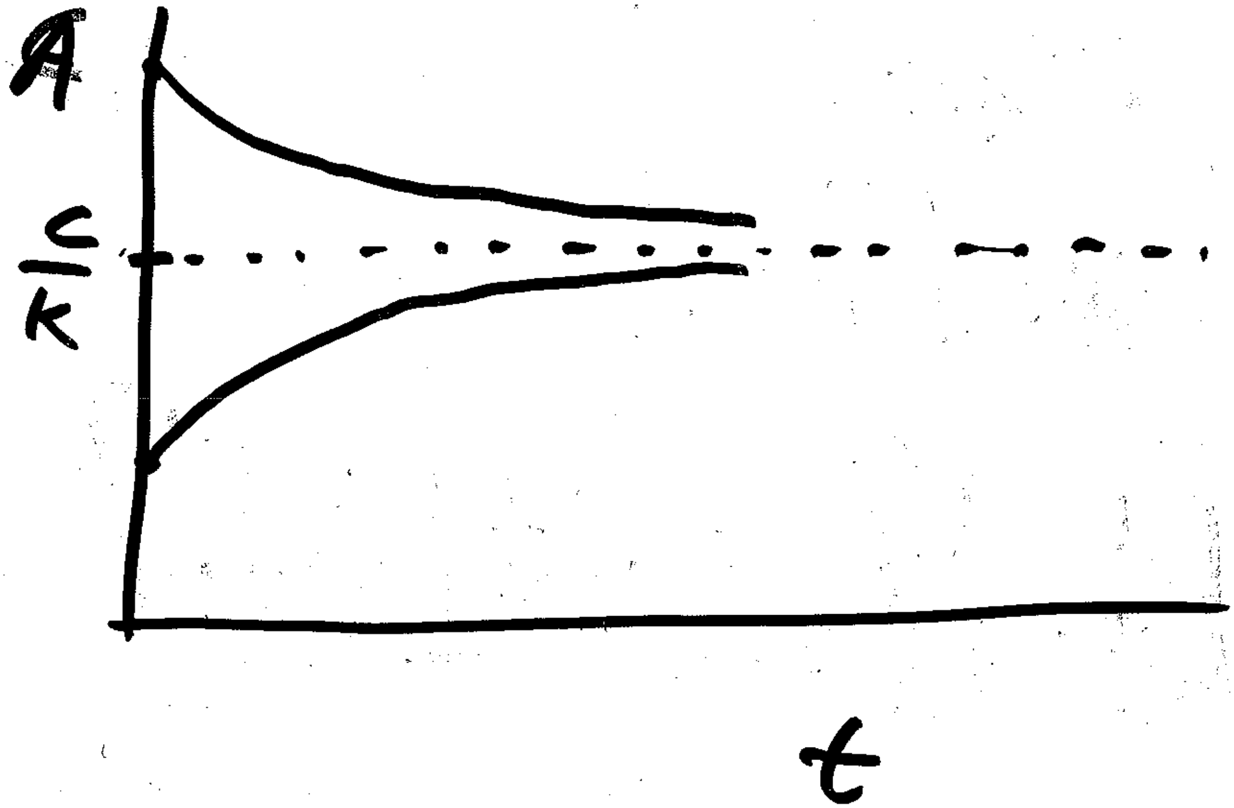
$$c_3 = C - kA_0$$

so

$$A = \frac{C}{k} - \left(\frac{C}{k} - A_0\right) e^{-kt}$$

or

$$A = \frac{c}{k} + \left(A_0 - \frac{c}{k}\right)e^{-kt}$$



Ex

Newton's law of cooling
The rate of change of temperature, T, of an object is proportional to the difference between T and the ambient temp. T_0.

$$\frac{dT}{dt} = -k(T - T_0), \quad k > 0$$

I.C. $T(0) = A$

separate, get

$$\frac{dT}{T - T_0} = -k\,dt$$

integrate, get

$$\ln|T - T_0| = -kt + c_1$$

or

$$T - T_0 = c_2 e^{-kt}$$

$T(0) = A$, so have

$$A - T_0 = c_2$$ so

$$T = T_0 + (A - T_0)e^{-kt}$$