Qualitative theory for 2/11/02

$$y' = F(y)$$

called an autonomous DE

If $F(y_s) = 0$, then we say y_s is a steady state.

<u>Ex</u>

$$y' = k(3 - y)$$

steady states satisfy

$$k(3-y) = 0 \text{ or } y = 3$$

steady states are solutions of the DE!

Suppose $F(y_s) = 0$

let $y(t) = y_s$

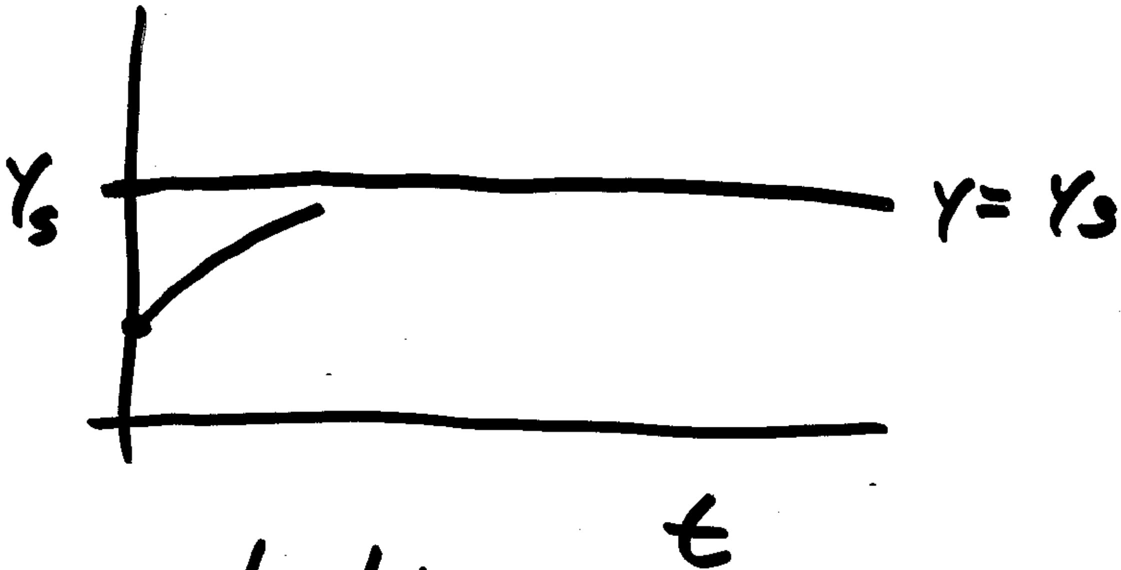
$y'(t) = 0$ and $F(y(t)) = F(y_s) = 0$

so it satisfies the DE.

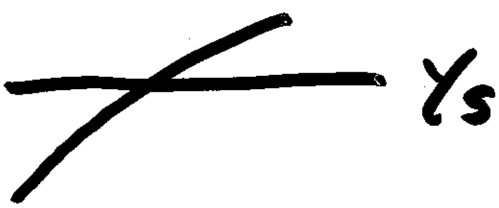
<u>Ex</u> Newton's law of cooling

$$\frac{dT}{dt} = -k(T - T_0)$$

steady state: $T = T_0$

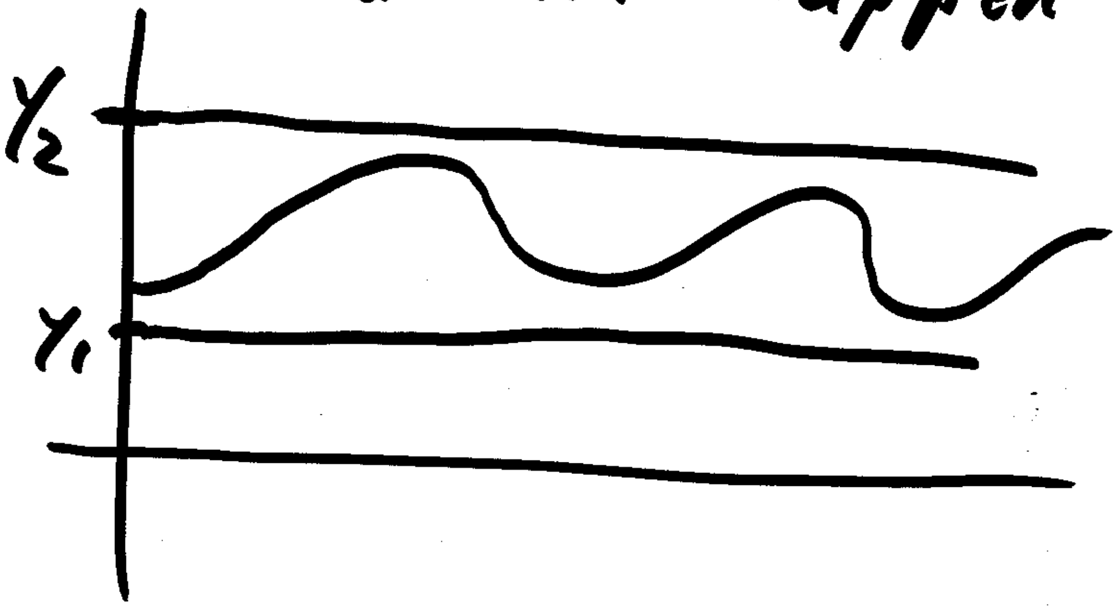


solution curves of $Y' = F(Y)$ can never cross, so can't have



Possible that $\lim_{t \to \infty} y(t) = Y_s$

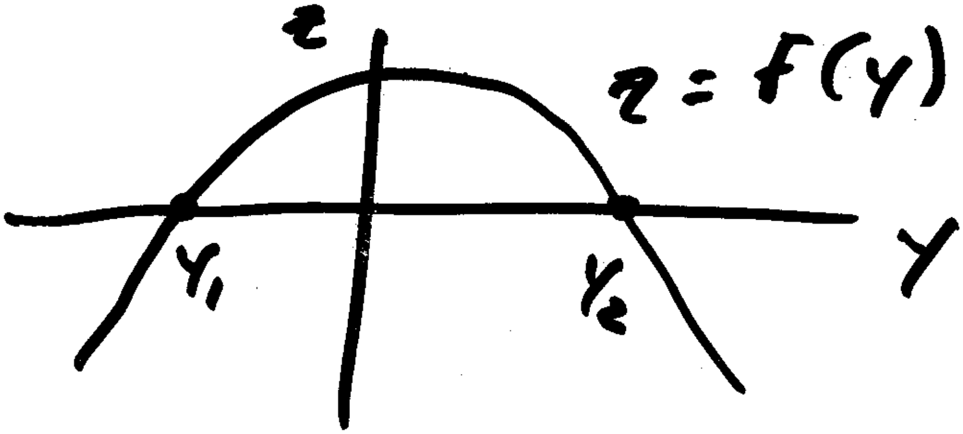
can this happen?



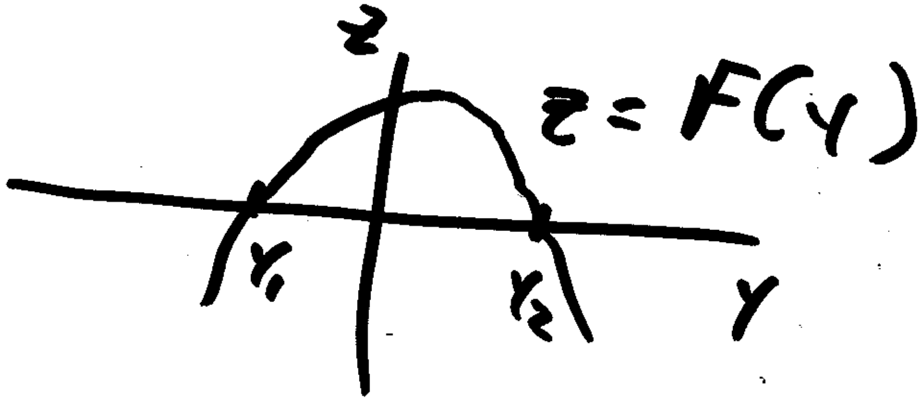
Y_1, Y_2 are steady states

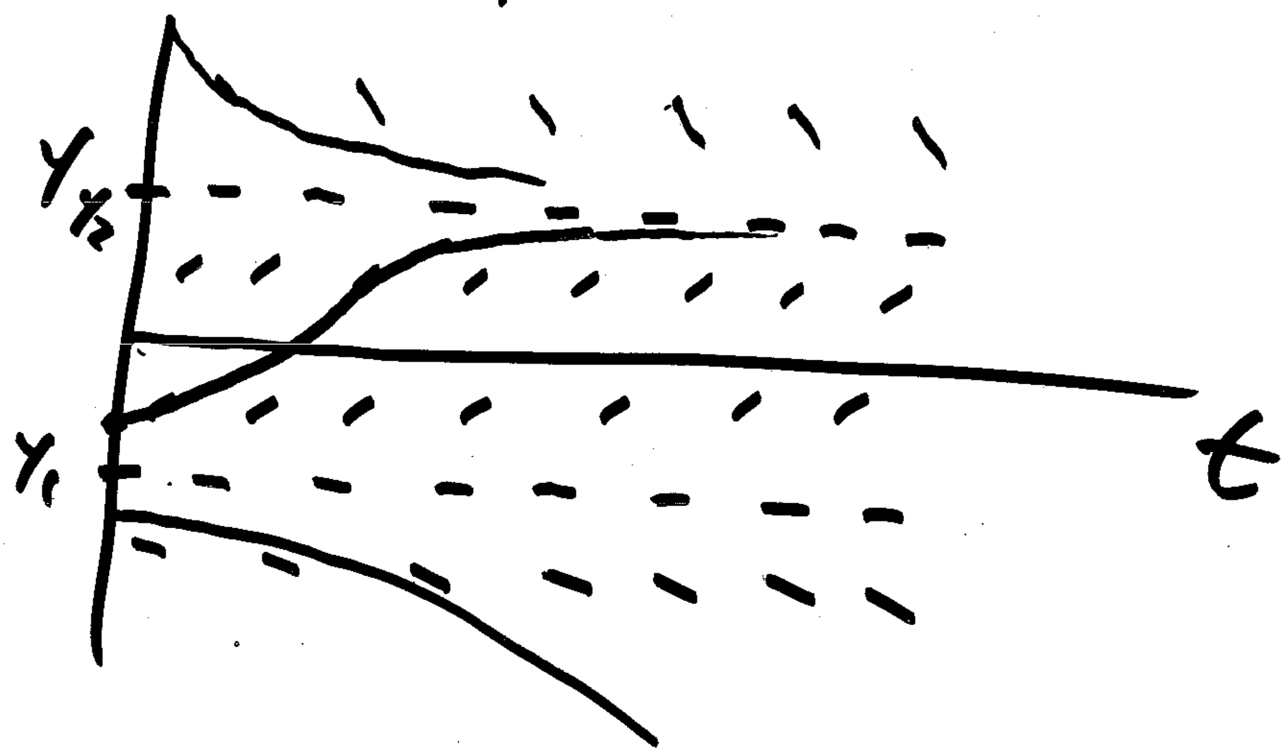
recall the DE

$$Y' = F(Y)$$



$$y' = F(y)$$

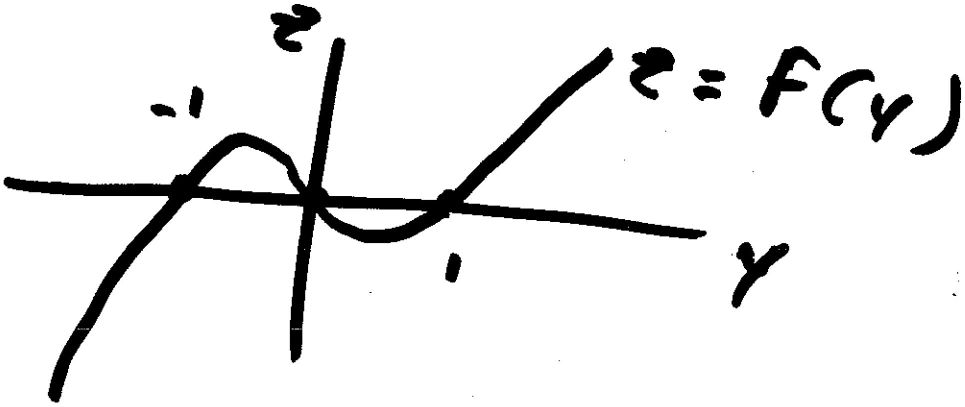




Ex

$$y' = y(y^2 - 1)$$

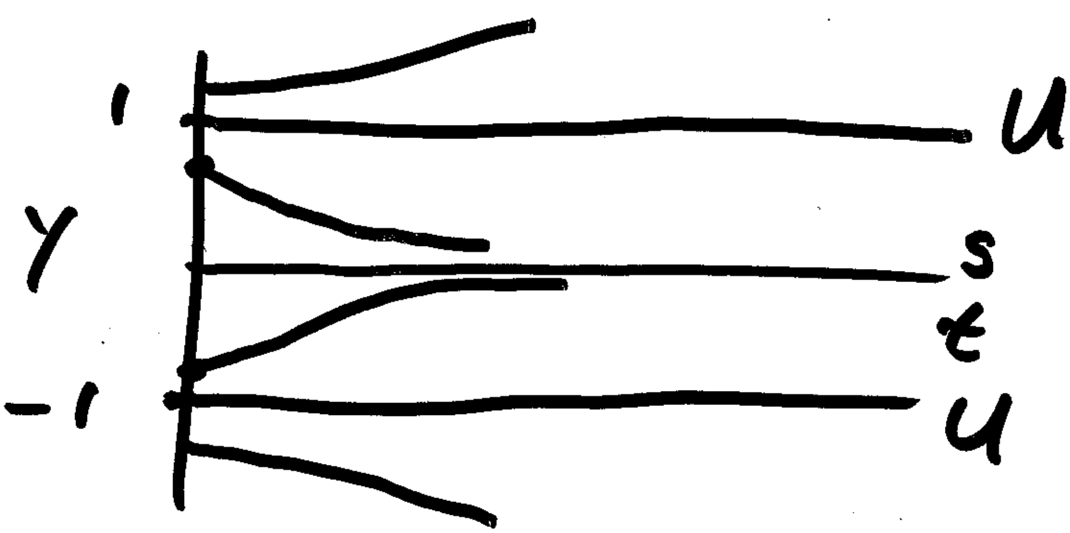
$$F(y) = y^3 - y$$



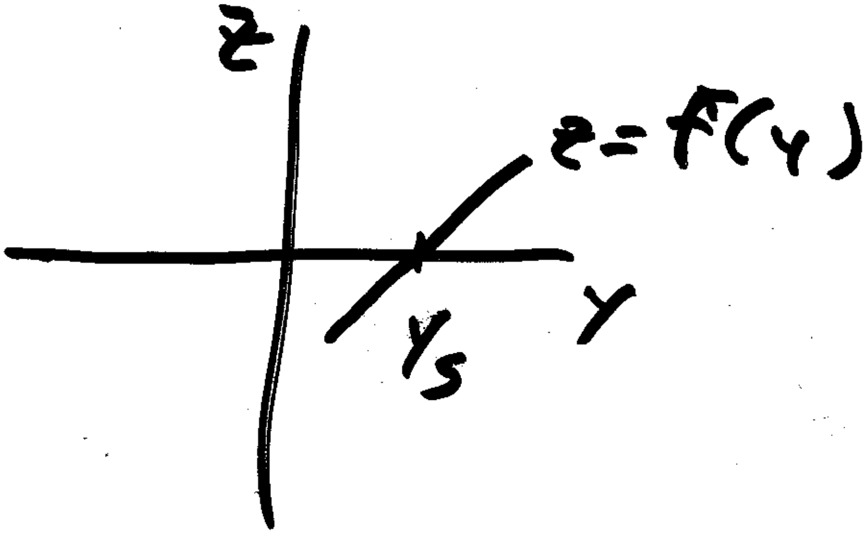
steady states: roots

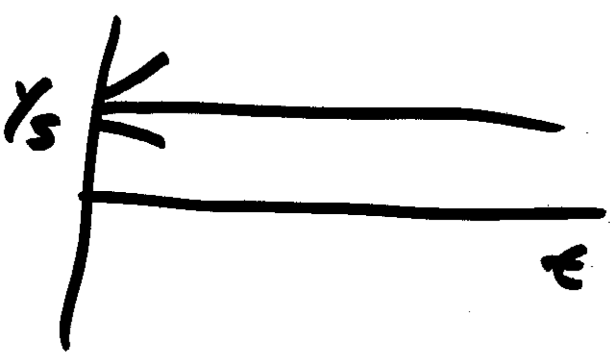
$F(y) = 0$ or $y(y^2-1) = 0$

$y = 0, \quad y = \pm 1$



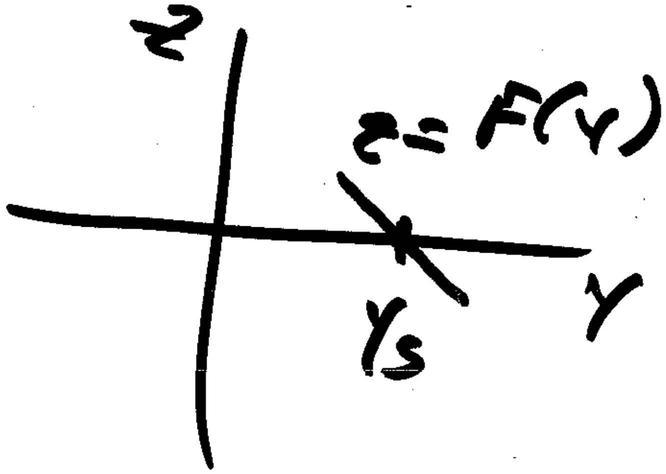
A steady state solution y_s of $y' = f(y)$ is asymptotically stable if all solutions with initial conditions near y_s satisfy $\lim_{t\to\infty} y(t) = y_s$. If y_s is not asymptotically stable we say it is unstable.

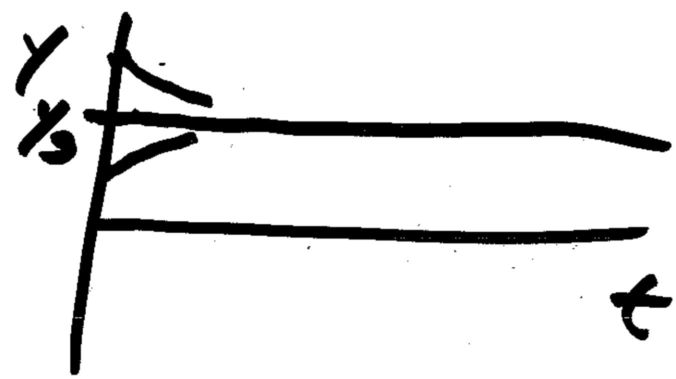




If $f(y) < 0$ for $y < y_s$ and $f(y) > 0$ for $y > y_s$, y_s is unstable

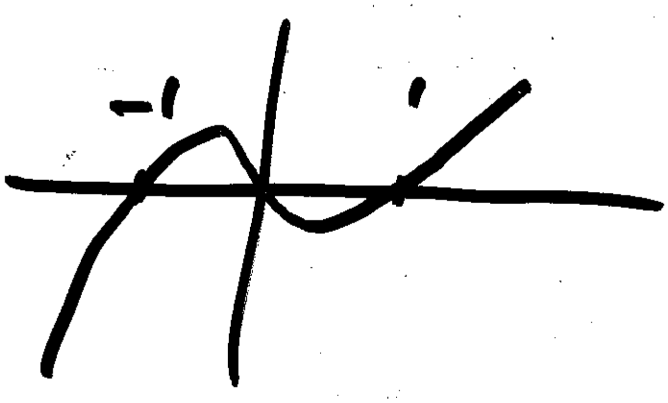
If $F(y) > 0$ for $y < y_s$ and $F(y) < 0$ for $y > y_s$, y_s is asymptotically stable.





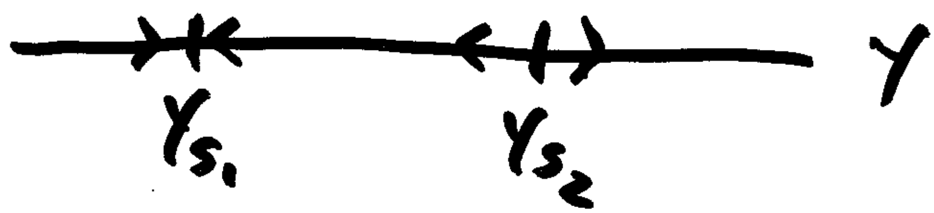
Ex

$$y' = y^3 - y$$



$y = -1$ U
$y = 0$ S
$y = 1$ U

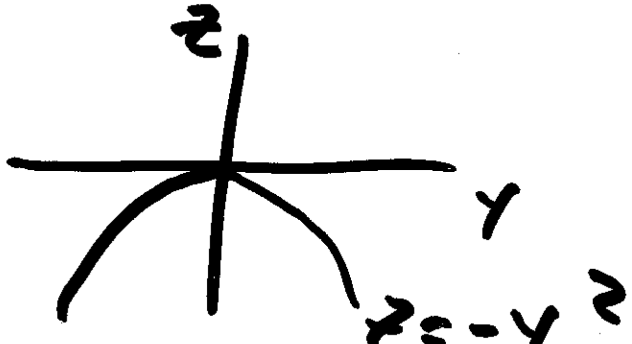
Summarizing behavior with the phase line.

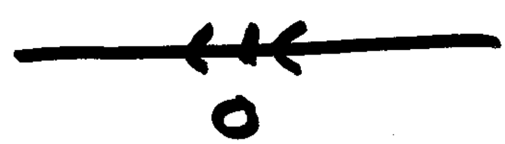


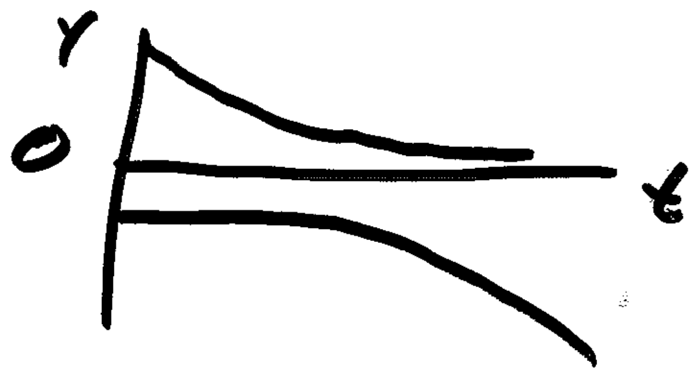
<u>Ex</u>

$$y' = -y^2$$

ss: $-y^2 = 0$, $y = 0$



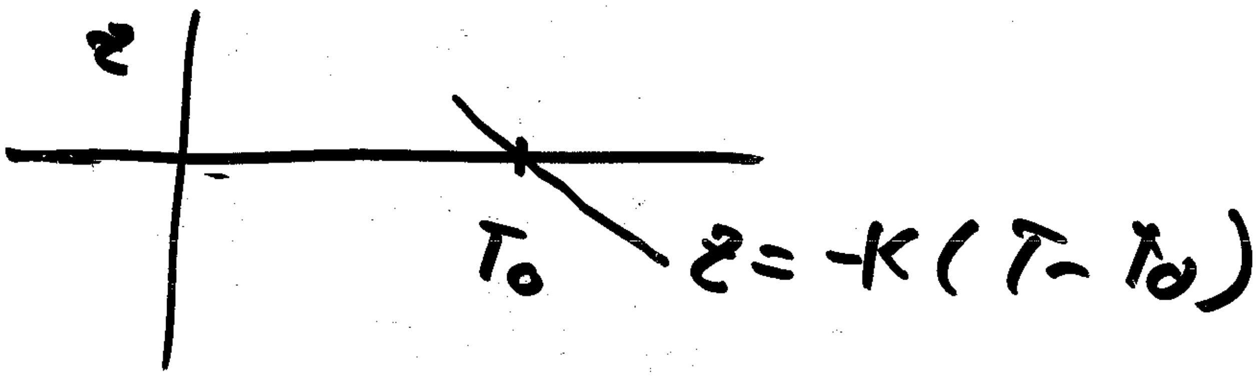




is $y = 0$
asy. stable
or unstable?

Ex Newton's law of cooling

$$\frac{dT}{dt} = -K(T - T_0)$$



is T_0 asy. stable?