Systems of differential equations.

- Needed for modeling competition between organisms, or predator-prey relationships.

A (two dim.) system has the form

$$x' = F(x, y)$$
$$y' = G(x, y)$$

Ex

$$x' = y$$
$$y' = -x$$

Fox + rabbits

$$\frac{dR}{dt} = R(k_R - b_R R - \beta F)$$

$$\frac{dF}{dt} = -F(k_F - \alpha R)$$

A solution to

$$x' = F(x,y)$$
$$y' = G(x,y)$$

is a pair of functions $x(t), y(t)$ that satisfy the DE.

Ex

$$x' = y$$
$$y' = -x$$

$x = A\sin(t)$, $y = A\cos(t)$

is a solution for any B.

To check, just plug into DE

$$x' = A\cos(t) \stackrel{?}{=} y \quad \text{yes}$$

$$y' = -A\sin(t) \stackrel{?}{=} -x \quad \text{yes}$$

both eq. must be satisfied.

How about $x = B\cos(t)$
$y = -B\sin(t)$?

consider

$x = A\sin(t)$, $y = A\cos(t)$

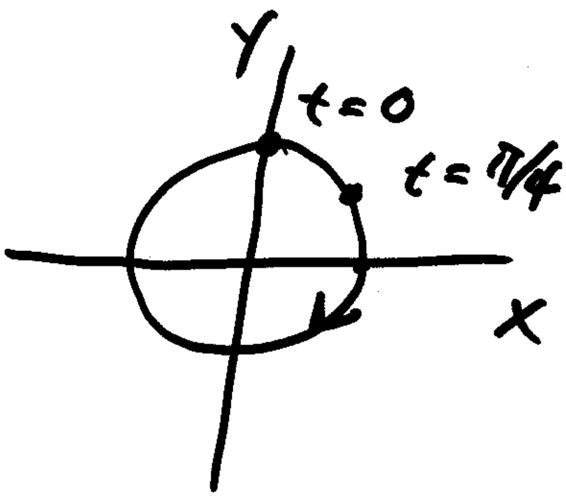
parametric curve

Note: $x^2 + y^2 = A^2\sin^2(t) + A^2\cos^2(t)$

$$= A^2(\sin^2(t) + \cos^2(t))$$

$$= A^2$$

graph is a circle of radius A



$x(0) = 0$

$y(0) = A$

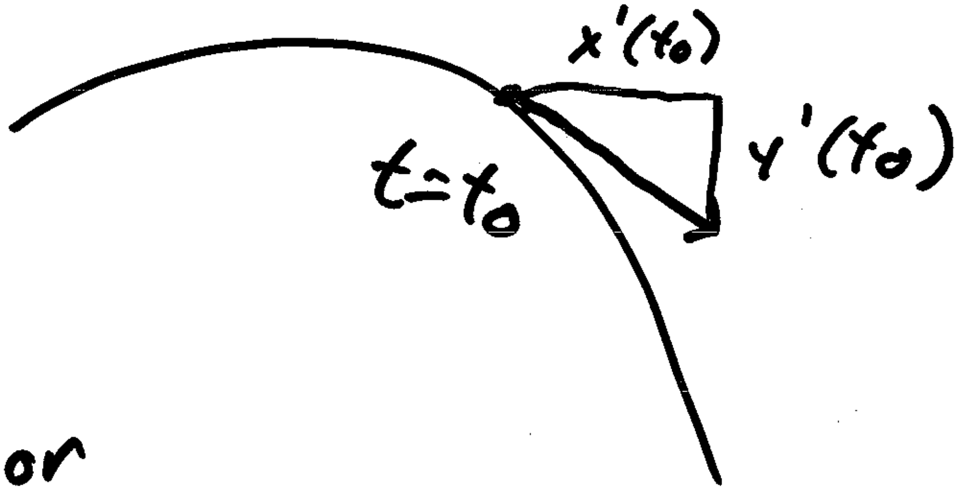
$x(\frac{\pi}{4}) = A\frac{\sqrt{2}}{2}$

$y(\pi/4) = A\frac{\sqrt{2}}{2}$

motion on the circle.

Tangents to parametric curves

$$x = f(t)$$
$$y = g(t)$$



or

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Ex

$x = \sin(t)$

$y = \cos(t)$

at $t = \frac{\pi}{4}$, what is $\frac{dy}{dx}$?

$$\frac{dx}{dt} = \cos(t)$$

$$\frac{dy}{dt} = -\sin(t)$$

$$\frac{dy}{dx} = \frac{-\sin(t)}{\cos(t)}$$

at $t = \pi/4$

$$\frac{dy}{dx} = -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

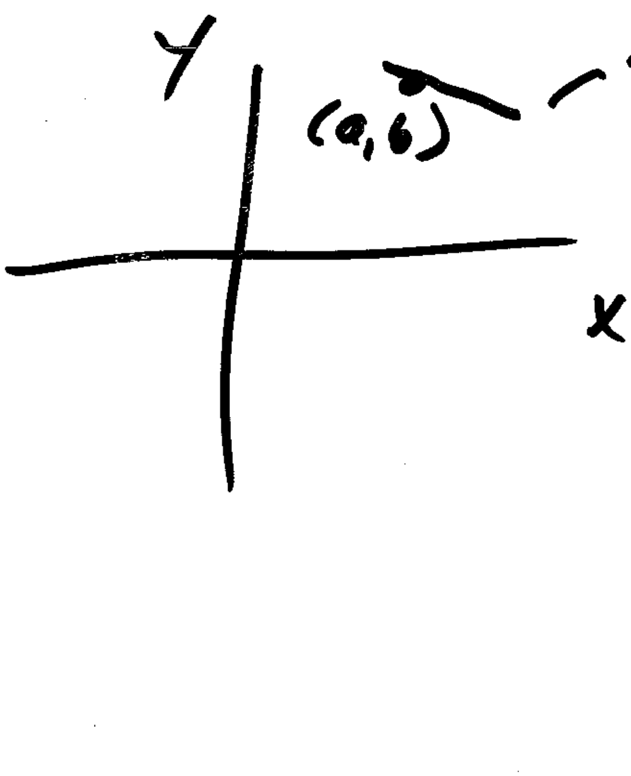
slope field for

$x' = F(x, y)$

$y' = G(x, y)$

$x' = y$

$y' = -x$



$\text{slope} = \dfrac{y'}{x'} = \dfrac{-a}{b}$

Gen'l lin. system

$$x' = ax + by$$
$$y' = cx + dy$$

Special cases

$$x' = ax$$
$$y' = dy$$

Solutions: $x = Ae^{at}, y = Be^{dt}$

i. a, d negative

$\lim_{t \to \infty} x(t) = 0, \lim_{t \to \infty} y(t) = 0$

ii. $a > 0, d < 0$

$x(t) \to \infty, y(t) \to 0$