MA1033 Theoretical Calculus III Name Solutions

Quiz 1 **A Term, 2017**

High: 20
Median: 20
Low: 17

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please determine whether each of the following sequences $\{a_n\}$ converges to a limit L or diverges. If it converges, please give the limit, L.

1. $a_n = \dfrac{1-n^2}{2+3n^2} = \dfrac{\frac{1}{n^2}-1}{\frac{2}{n^2}+3}$ (+2) (+1)

So $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{\frac{1}{n^2}-1}{\frac{2}{n^2}+3} = -1/3$ (+2)

$-1/3$

2. $a_n = \dfrac{\sin^2 n}{\sqrt{n}}$ (+2)

Since $\forall n, 0 \le |\sin^2 n| \le 1$, by the Squeeze Thm, (+1)

$0 \le \lim_{n\to\infty} \dfrac{\sin^2 n}{\sqrt{n}} \le \lim_{n\to\infty} \dfrac{1}{\sqrt{n}} = 0 \Rightarrow \lim_{n\to\infty} a_n = 0.$ (+2)

0

3. $a_n = \dfrac{\ln(n-1)^2}{\ln(n+1)}$ l'Hôpital's Rule

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{\ln(n-1)^2}{\ln(n+1)} = 2 \lim_{n\to\infty} \dfrac{\ln(n-1)}{\ln(n+1)} \overset{\text{l'H}}{=} 2 \lim_{n\to\infty} \dfrac{\frac{1}{n-1}}{\frac{1}{n+1}}$ (+2) (+2)

$= 2 \lim_{n\to\infty} \dfrac{n+1}{n-1} = 2$ (+1)

2

4. $a_n = (e^n+1)^{1/n}$

$\ln(L) = \ln\left(\lim_{n\to\infty} a_n\right) \overset{(+1)}{=} \lim_{n\to\infty} \ln(a_n) = \lim_{n\to\infty} \dfrac{\ln(e^n+1)}{n} \overset{\text{l'H}}{=} \lim_{n\to\infty} \dfrac{\frac{e^n}{e^n+1}}{1}$ (+1) (+1)

$= \lim_{n\to\infty} \dfrac{1}{1+e^{-n}} = 1 \Rightarrow L = e$ (+1)

e