MA1033 Theoretical Calculus III Name *Solutions*

High: 20
Median: 20
Low: 17

Quiz 5 **A Term, 2017**

(5 points each) For each power series, please find x_o, the center of the interval of convergence, and R, the radius of convergence. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3(x+1)^n}{3^n}$

$$R = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|} = \lim_{n\to\infty} \frac{n^3}{\cancel{3^n}} \frac{3^{n+1}}{(n+1)^3} = 3 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^3 = 3$$

+1 +1 +1

$x_o =$ -1 +1 $R =$ 3 +1

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 4^n x^n}{n \ln n}$

$$R = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|} = \lim_{n\to\infty} \frac{4^n}{n \ln n} \cdot \frac{(n+1)\ln(n+1)}{4^{n+1}} = \frac{1}{4} \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{\ln(n+1)}{\ln(n)} = \frac{1}{4}$$

+1 +1 +1

$x_o =$ 0 +1 $R =$ $\frac{1}{4}$ +1

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\, x^n}{(3n-2)!!!}$

$$R = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|} = \lim_{n\to\infty} \frac{\cancel{n!}}{(3n-2)!!!} \cdot \frac{(3(n+1)-2)!!!}{(n+1)!} = \lim_{n\to\infty} \frac{(3n+1)\cancel{(3n-2)!!!}}{\cancel{(3n-2)!!!}\,(n+1)}$$

$$= \lim_{n\to\infty} \frac{3n+1}{n+1} = 3$$

+1 +1 +1 +1 +1

$x_o =$ 0 $R =$ 3

4. Suppose $\displaystyle\sum_{k=0}^{\infty} a_k x^k$ converges with radius of convergence $R_a = 2$, while $\displaystyle\sum_{k=0}^{\infty} b_k x^k$ converges with radius of convergence $R_b = 3$. What's the radius of convergence for the series $\displaystyle\sum_{k=0}^{\infty} \frac{b_k}{a_k 5^k} x^k$? Assume that $a_k \neq 0\, \forall k$.

$$R = \lim_{k\to\infty} \frac{b_k}{a_k 5^k} \cdot \frac{a_{k+1} 5^{k+1}}{b_{k+1}} = 5 \left(\lim_{k\to\infty} \frac{b_k}{b_{k+1}}\right)\left(\lim_{k\to\infty} \frac{a_{k+1}}{a_k}\right) = 5 R_b \left(\frac{1}{R_a}\right)$$

$$= \frac{15}{2}$$

+2 +1 +1 +1