MA1033 Theoretical Calculus III Name *Solutions*

Quiz 6 **A Term, 2017**

Show all work needed to reach your answers. For questions #1 and #2, please find a power series representation for each function, then for #1, give the radius of convergence.

1. (6 points) $f(x) = x^2 e^{-3x}$ $= x^2\left(\sum_{K=0}^{\infty} \frac{(-3x)^K}{K!}\right)$ (+4)

$= x^2 \sum_{K=0}^{\infty} \frac{(-3)^K x^K}{K!}$

(+1) Series: $\sum_{K=0}^{\infty} \frac{(-3)^K x^{K+2}}{K!}$ (+1) $R =$ $+\infty$

2. (6 points) $f(x) = \int_0^x \frac{1}{1-t^2}dt$ $= \int_0^x \sum_{K=0}^{\infty} (t^2)^K dt$ (+3) $= \sum_{K=0}^{\infty} \int_0^x t^{2K} dt$

$= \sum_{K=0}^{\infty} \frac{t^{2K+1}}{2K+1}\Big|_0^x$ (+2)

(+1) Series: $\sum_{K=0}^{\infty} \frac{x^{2K+1}}{2K+1}$

3. (8 points) If one approximates $\cos(x/3)$ on the interval $[-1, 1]$ by $1 - \frac{(x/3)^2}{2} = 1 - x^2/18$ (the first two terms of the Taylor series), what is a bound on the error ? *Explain.*

Because the Taylor series for cos is an alternating series, (+2) the error is bounded by (+2) the next term in the series: $|E| \le \left|\frac{x^4}{3^4 \cdot 4!}\right|$ (+3). *Since* $|x| \le 1$ *on* $[-1,1]$,

$|E| \le \frac{1}{3^4 4!} = \frac{1}{2^3 3^5} = \frac{1}{1944}$ (+1) $\simeq 0.0005$