High: 20
Median: 20
Low: 10

MA1033 Theoretical Calculus III Name Solutions

Quiz 3 **A Term, 2017**

Show all work needed to reach your answers.

(5 points each) For each series, please state what type it is (harmonic, alternating or geometric), whether it converges or diverges (circle one), and if possible what limit L the series converges to.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{\sqrt{n^2+1}}$

In the n^{th} Term Test, since (+1)

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} \overset{(+3)}{=} \lim_{n\to\infty} \frac{1}{\sqrt{1+1/n^2}} = 1$, this series must diverge. (+1)

converges **diverges** $L =$

2. $\sum_{n=1}^{\infty} \sin^n 1$

Since $0 < \sin 1 < 1$ (+1), this geometric series (+1) must converge:
Here $a = \sin 1$ (+1) and $r = \sin 1$ (+1), so $L = \frac{\sin 1}{1-\sin 1}$

converges diverges $L = \frac{\sin 1}{1-\sin 1}$ (+1)

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

This is the alternating harmonic series (or its negative), so by the alternating series test (+1) (sign alternates (+1), $a_n \searrow 0$ (+2)), this series must converge, though we can't easily give the exact value

converges (+1) diverges $L =$

4. $\sum_{n=0}^{\infty} \frac{1-2^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$ (+1) $- \sum_{n=0}^{\infty} \left(2/3\right)^n$ (+1) provided both of the latter series converge. Since both are geometric (+1) with $r<1$,

$$\frac{1}{1-1/3} - \frac{1}{1-2/3} = \frac{3}{3-1} - \frac{3}{3-2} = \frac{3}{2} - 3 = -3/2$$

converges diverges $L = -3/2$ (+2)