MA1033 Theoretical Calculus III   Name *Solutions*

Quiz 3   A Term, 2018

High: 20
Median: 19
Low: 15

Show all work needed to reach your answers.

(5 points each) For each series, please state what type it is (harmonic, alternating, geometric, etc.), whether it converges or diverges (circle one), and if possible to which limit L it converges.

1. $\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{1}{n}\right)$

Alternating Series (+2) ✓

Notice that $n < n+1 \Leftrightarrow \frac{1}{n+1} < \frac{1}{n}$. Then since $\sin\frac{1}{n}$ is increasing for $0 < \frac{1}{n} < \frac{\pi}{2}$, $\sin\left(\frac{1}{n+1}\right) < \sin\left(\frac{1}{n}\right)$ $\forall n \in \mathbb{Z}^+$, by the Liebniz Alternating Series Test, this series converges. (+2)

(+1) ✓ converges   diverges

$L = \boxed{-\sin(1) < L < 0}$

2. $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \ldots + \frac{2^{n-1}}{3^n} + \ldots$

Geometric Series (+1)

$a = 1/3$, $r = 2/3$ (+1)

Since $r < 1$

$L = \frac{\frac{1}{3}}{1 - 2/3} = \frac{1/3}{1/3} = 1$ (+1)

(+1) converges   diverges

$L = 1$ (+1)

3. $\sum_{n=1}^{\infty}\frac{(-1)}{n} = -\sum_{n=1}^{\infty}\frac{1}{n}$ (+1)

This is the negative of the harmonic series. (+2)

converges   diverges (+2)

$L =$ DNE

4. $\sum_{n=1}^{\infty}\sqrt[n]{2} = \sum_{n=0}^{\infty} 2^{1/n}$ This series is not geometric. (+1)

Since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 2^{1/n} \overset{(+2)}{=} 2^{\lim 1/n} = 2^0 = 1$ by the nth term test, this series diverges.

converges   diverges (+2)

$L =$ DNE