Quiz 4 High: 20 Median: 20 Low: 14 A Term, 2017

(5 points each) For each series, please state whether it converges or diverges (circle one), which test(s) you use to reach your conclusion, and if possible what limit L the series converges to. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \arctan n}{\sqrt{n}}$ As $n \to \infty$, $\arctan n \to \frac{\pi}{2}$ (+1), so $a_n = \frac{\arctan n}{\sqrt{n}} \searrow 0$. (+1)
Since also the sign of the n^{th} term alternates (+1), by the Leibniz alternating series (+1) test, this series must converge.

(+1) converges diverges $L =$ ______

2. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$ Since $\frac{n}{n^4+1} < \frac{n}{n^4} = \frac{1}{n^3}$ (+2), this series converges by comparison (+2) with $\sum_{n=1}^{\infty} \frac{1}{n^3}$ (a p-series). One can also use the limit comparison test or the integral test.

(+1) converges diverges $L =$ ______

3. $\sum_{k=0}^{\infty} e^{-k}$ $= \frac{1}{1-1/e} = \frac{e}{e-1}$ (+2) since this is a geometric series (+1) with $r = e^{-1} < 1$ (+1). One can also use the integral test or the ratio test, but these don't give the value L.

(+1) converges diverges $L = \frac{e}{e-1}$

4. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ Applying the integral test (+1): $\int_2^{\infty} \frac{dx}{x(\ln x)^2}$ (+1) $= \int_{\ln(2)}^{\infty} \frac{du}{u^2}$ (+1) $= -u^{-1}\Big|_{\ln(2)}^{\infty}$ (+1) $= \frac{1}{\ln(2)}$
Since this integral converges, so does this series.

(+1) converges diverges $L =$ ______