MA1033 Theoretical Calculus III Name Solutions

Quiz 5 A Term, 2018

High: 20
Median: 20
Low: 16

(5 points each) For each power series, please find x_o, the center of the interval of convergence, and R, the radius of convergence. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-\overset{x_o}{3})^n}{n^2}$

$R \overset{+1}{=} \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| \overset{+1}{=} \lim_{n\to\infty} \frac{\cancel{2^n}}{n^2}\cdot\frac{(n+1)^2}{2^{n+1}} \overset{+1}{=} \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 = 1/2$ +1

$x_o = 3$ +1 $R = 1/2$

2. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n 4^n x^n}{n\ln n}$

$R \overset{+1}{=} \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{4^n (n+1)\ln(n+1)}{4^{n+1}\, n\ln(n)}$ +1 $= \frac{1}{4}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\lim_{n\to\infty}\frac{\ln(n+1)}{\ln n}$ +1

$\underset{\text{l'Hôpital}}{=} \frac{1}{4}\lim_{n\to\infty}\frac{n}{n+1} = \frac{1}{4}\lim_{n\to\infty}\left(\frac{1}{1+1/n}\right)$ +1

$x_o = 0$ +1 $R = 1/4$

3. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-\overset{x_o}{1})^n}{3^{n/2}}$

$R \overset{+1}{=} \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\frac{3^{\frac{n+1}{2}}}{3^{n/2}}$ +1 $= \lim_{n\to\infty}\frac{\cancel{3^{n/2}}\,3^{1/2}}{\cancel{3^{n/2}}}$ +1 $= \sqrt{3}$ +1

$x_o = 1$ +1 $R = \sqrt{3}$

4. Suppose $\displaystyle\sum_{k=0}^{\infty} a_k x^k$ converges with radius of convergence $R = 7$. What's the radius of convergence for the series $\displaystyle\sum_{k=0}^{\infty}\frac{3a_k}{k!}x^k$?

$7 = R \overset{+1}{=} \lim_{k\to\infty}\left|\frac{a_k}{a_{k+1}}\right|$

For the second series,

$R \overset{+1}{=} \lim_{k\to\infty}\left|\frac{a_k}{a_{k+1}}\right| \overset{+1}{=} \lim_{k\to\infty}\left|\frac{\cancel{3}\,a_k\,(k+1)\cancel{!}}{\cancel{3}\,a_{k+1}\,\cancel{k!}}\right| \overset{+1}{=} \lim_{k\to\infty}\left|\frac{a_k}{a_{k+1}}\right|\cdot\lim_{k\to\infty}\frac{k+1}{1}$

$= 7\cdot(+\infty) = +\infty$ So for the latter series, $R = +\infty$. +1