MA1033 Theoretical Calculus III Name Solutions

Quiz 6 A Term, 2018

Show all work needed to reach your answers. For questions #1 and #2, please find a power series representation for each function, then for #1, give the radius of convergence. For questions #3 and #4, please compute the indicated limit.

High: 20
Median: 19
Low: 13

1. (5 points) $f(x) = \dfrac{\ln(1+x)}{x} = \dfrac{1}{x}\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{k}x^k$ (+2)

Series: $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{k}x^{k-1}$ (+2) $R =$ 1 (+1)

2. (5 points) $f(x) = \displaystyle\int_0^x \frac{1-e^{-t^2}}{t}dt = \int_0^x \frac{1-\sum_{k=0}^{\infty}\frac{(-t^2)^k}{k!}}{t}dt$ (+1) $= \displaystyle\int_0^x \frac{\cancel{1}-\left(\cancel{1}+\sum_{k=1}^{\infty}\frac{(-1)^k t^{2k}}{k!}\right)}{t}dt$ (+1)

$= -\displaystyle\int_0^x \sum_{k=1}^{\infty}\frac{(-1)^k}{k!}t^{2k-1}dt = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}\int_0^x t^{2k-1}dt$ (+1) $= \displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}\frac{x^{2k}}{(2k)}$ (+1)

$\boxed{j=k-1}$

Series: $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}\frac{x^{2k}}{(2k)} = \sum_{j=0}^{\infty}\frac{(-1)^j}{(j+1)!}\frac{x^{2j+2}}{(2j+2)}$ (+1)

3. (5 points) $\displaystyle\lim_{x\to 0}\frac{1+x-e^x}{x^2} = \lim_{x\to 0}\left(\frac{1+x-\sum_{k=0}^{\infty}\frac{x^k}{k!}}{x^2}\right)$ (+1)

$= \displaystyle\lim_{x\to 0}\frac{\cancel{1}+\cancel{x}-(\cancel{1}+\cancel{x}+\frac{1}{2}x^2+\frac{1}{6}x^3+O(x^4))}{x^2}$ (+1) (+1)

$= \displaystyle\lim_{x\to 0}\frac{-\frac{1}{2}\cancel{x^2}-\frac{1}{6}x^{\cancel{3}}+O(x^{\cancel{4}\,2})}{\cancel{x^2}} = -1/2$ (+1) (+1)

Limit: $-1/2$

4. (5 points) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right) = \lim_{x\to 0}\left(\frac{\sin x - x}{x\sin x}\right)$ (+1)

$= \displaystyle\lim_{x\to 0}\frac{\cancel{x}-\frac{x^3}{6}+O(x^5)-\cancel{x}}{x^2-\frac{x^4}{6}+O(x^6)}$ (+2) $= \displaystyle\lim_{x\to 0}\frac{x/6+O(x^3)}{1-\frac{x^2}{6}+O(x^4)} = 0$ (+1) (+1)

Limit: 0