MA143X Theoretical Calculus III Name Solutions

Quiz 2 A Term, 2013

High: 20
Median: 18
Low: 5

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please determine whether each of the following sequences $\{a_n\}$ converges to a limit L or diverges. If it converges, please give the limit, L.

1. $a_n = (-1)^n \dfrac{n}{1+n}$

Since $a_{2n} = \frac{2n}{2n+1} \to 1$
while $a_{2n-1} = -\frac{2n-1}{2n} \to -1$,
this limit DNE.

Diverges

2. $a_n = \sqrt{n}(\sqrt{n+2} - \sqrt{n-1})$ $\cdot \frac{(\sqrt{n+2}+\sqrt{n-1})}{(\sqrt{n+2}+\sqrt{n-1})}$ ‖necessary

$= \dfrac{\sqrt{n}\,(n+2-(n-1))}{\sqrt{n+2}+\sqrt{n-1}}$ ⊖1 or ⊖2 for algebraic mistakes.

$= \dfrac{3}{\sqrt{1+2/n}+\sqrt{1-1/n}} \to \dfrac{3}{2}$

3/2

3. $a_n = \left(\dfrac{1}{n}\right)^{1/n}$

$\lim_{n\to\infty} a_n = e^{\lim_{n\to\infty} \frac{1}{n}\ln(\frac{1}{n})}$ +2 $= e^{\lim_{n\to\infty} \frac{\ln(1/n)}{n}}$

$= e^{\lim_{n\to\infty} \frac{\frac{1}{1/n}\cdot(-\frac{1}{n^2})}{1}} = e^{\lim_{n\to\infty} -\frac{1}{n}} = e^0$ +1

↑ l'Hôpital's Rule +2

1

4. $a_n = \dfrac{n\sin(\pi n)}{3n^2+4} = 0 \quad \forall n$

since $\sin(\pi n) = 0 \ \forall n$

0