MA143X Theoretical Calculus III Name Solutions

Quiz 4 A Term, 2013

High: 20
Median: 17
Low: 6

(5 points each) For each series, please state whether it converges or diverges (circle one), and which test(s) you use to reach your conclusion. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\sum_{n=1}^{\infty} 2^{-(2/n^2)}$

By the n^{th} term test (+1)

$a_n = 2^{-(2/n^2)} \to 1 \neq 0$ as $n \to +\infty$. (+1) (+2)

Hence ...

(+1) converges **diverges**

2. $\sum_{n=1}^{\infty} \frac{5^n}{n^n}$

Notice that for $n \geq 6$, $\frac{5^n}{n^n} < \left(\frac{5}{6}\right)^n$ (+2) So this series converges by Comparison (+1) with the geometric series (+1) $\sum_{n=1}^{\infty} (5/6)^n$

Using the Ratio Test (+1) since

$\lim_{n\to\infty} \frac{5^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{5^n}$ (+1) $= 5 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1}$

(+1) $= \frac{5}{e} \lim_{n\to\infty} \frac{1}{n+1}$ (+1) $= 0$, this series ...

(+1) **converges** diverges

3. $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n + 1}$

Notice that $\frac{2^n}{3^n+1} < \left(\frac{2}{3}\right)^n$ (+2). So this series absolutely converges by comparison (+1) with the geometric (+1) series $\sum_{n=1}^{\infty} (2/3)^n$

Since this is an alternating series with $a_n = \frac{2^n}{3^n+1}$ (+1) $\searrow 0$ (+2), by the Alternating Series Test (+1), this series ...

(+1) **converges** diverges

4. $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$

Here $a_n = \frac{\sqrt{n+1} - \sqrt{n}}{n} \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$ (+1) $= \frac{n+1-n}{n(\sqrt{n+1} + \sqrt{n})} < \frac{1}{2n^{3/2}}$ (+1)

So this series converges by comparison (+1) with the p-series (+1) $\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $(p = 3/2 > 1)$ (+1)

converges diverges