MA143X Theoretical Calculus III Name Solutions

Quiz 5 A Term, 2013

(5 points each) For each power series, please find a, the center of the interval of convergence, and R, the radius of convergence. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

High: 20
Median: 19
Low: 16

1. $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^3}$

$$\frac{1}{R} = L \overset{+2}{=} \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)^3} \cdot \frac{n^3}{\cancel{3^n}} \overset{+1}{=} 3 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^3 \overset{+1}{=} 3$$

$a = \underline{0}$ (+1) $R = \underline{1/3}$

2. $\sum_{n=1}^{\infty} \frac{n!}{2^n}(x-5)^n$

$$\frac{1}{R} = L \overset{+2}{=} \lim_{n\to\infty} \frac{\cancel{(n+1)!}}{2^{n+1}} \cdot \frac{\cancel{2^n}}{\cancel{n!}} \overset{+1}{=} \frac{1}{2} \lim_{n\to\infty} n+1 \overset{+1}{=} +\infty$$

$a = \underline{5}$ (+1) $R = \underline{0}$

3. $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}(x+7)^n$

$$\frac{1}{R} = L \overset{+1}{=} \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(2(n+1))!} \cdot \frac{(2n)!}{n^n} \overset{+1}{=} \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n \cdot \frac{\cancel{(n+1)}}{1} \cdot \frac{\cancel{(2n)!}}{\cancel{(2n+2)}(2n+1)\cancel{(2n)!}}$$

$$\overset{+1}{=} \frac{1}{2} \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n \cdot \lim_{n\to\infty} \frac{1}{2n+1} = \frac{1}{2} \cdot e \cdot \frac{1}{\infty} \overset{+1}{=} 0$$

$a = \underline{-7}$ (+1) $R = \underline{+\infty}$

4. Consider the power series $\sum_{k=1}^{\infty} a_k (x-a)^k$. Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$ where $0 < L < +\infty$.
Please describe the set of $x \in \mathbb{R}$ where this power series converges or diverges. Suggestion: draw a diagram.

This power series converges absolutely on a open (+2) interval of radius $R = \frac{1}{L}$ (+1) centered at $x = a$: $(a-R, a+R)$. The series diverges for either $x < a-R$ or (+2) $x > a+R$, and nothing in general can be said about the endpoints: $a \pm R$.