Quiz 6

A Term, 2013

(5 points each) Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value. For questions #1 and #2, please find a power series representation for each function, then give the radius of convergence.

$$Recall: e^{u} = \sum_{k=0}^{\infty} \sum_{k=0}^{\infty} K! \quad for \text{ all } u$$

$$Recall: e^{u} = \sum_{k=0}^{\infty} K! \quad for \text{ all } u$$

$$(R = +\infty)$$

$$= X^{2} \sum_{k=0}^{\infty} \frac{(-3x)^{k}}{K!}$$

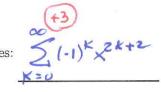
$$\frac{3}{(-3)^K \times K+2} \qquad R = + \infty$$

$$2. g(x) = \frac{x^2}{1+x^2}$$

$$Recoll: \frac{1}{1-u} = \sum_{k=0}^{\infty} u^k \text{ for } |u| \le 1$$

$$Here g(x) = X^2 \left(\frac{1}{1-(-x^2)}\right)$$

$$= X^2 \left(\sum_{k=0}^{\infty} (-x^2)^k\right)$$



$$R = 1$$

3. Please find a bound for the integral of the Taylor remainder 
$$R_{n+1}(x)$$
 for  $\int_0^1 \frac{\sin x}{x} dx$ 

For  $f(x) = \sin x$ ,  $R_{n+1}(x) = \frac{f(n)(x)}{(n+1)!} \times \frac{f(n)(x)}{(n+1)!} = \frac{f(n)(x)}{(n+1)!} \times \frac{f(n)(x)}{(n+1)!} \times$ 

4. Consider the power series  $\sum_{k=0}^{\infty} a_k (x-a)^k$ . If this power series equals f(x) for some  $f \in C^{\infty}$ , what are the values of the coefficients  $a_k$  in terms of f and its derivatives? Based on this power series, what power series represents f'(x).

$$a_k = \frac{\int_{-K}^{K} a}{K!}$$

$$f'(x) = \sum_{k=1}^{\infty} Ka_{k}(x-a)^{k-1} = \sum_{k=1}^{\infty} \frac{f'(k)}{(k-1)!} (x-a)^{k-1}$$

$$= \sum_{k=0}^{\infty} \frac{f''(a)}{k!} (x-a)^{k}$$