MA143X Theoretical Calculus III Name Solutions

Quiz 6 A Term, 2013

(5 points each) Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value. For questions #1 and #2, please find a power series representation for each function, then give the radius of convergence.

High	17
Median	14
Low	10

1. $f(x) = x^2 e^{-3x}$

Recall: $e^u = \sum_{k=0}^{\infty} \frac{u^k}{k!}$ (+1) for all u $(R = +\infty)$

Here $f(x) = x^2 e^{(-3x)}$
$= x^2 \sum_{k=0}^{\infty} \frac{(-3x)^k}{k!}$

Series: $\sum_{k=0}^{\infty} \frac{(-3)^k x^{k+2}}{k!}$ (+3) $R = +\infty$ (+1)

2. $g(x) = \dfrac{x^2}{1+x^2}$

Recall: $\frac{1}{1-u} = \sum_{k=0}^{\infty} u^k$ (+1) for $|u| \le 1$

Here $g(x) = x^2 \left(\frac{1}{1-(-x^2)}\right)$
$= x^2 \left(\sum_{k=0}^{\infty} (-x^2)^k\right)$

So Series: $\sum_{k=0}^{\infty} (-1)^k x^{2k+2}$ (+3) $R = 1$ (+1)

3. Please find a bound for the integral of the Taylor remainder $R_{n+1}(x)$ for $\int_0^1 \frac{\sin x}{x} dx$ (+1)

For $\tilde{f}(x) = \sin x$, $\tilde{R}_{n+1}(x) = \frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}$ (+1). Since $|f^{(n+1)}(c)| \le 1$ $\forall c, n$,

for $\frac{\sin x}{x}$, $\left|\int_0^1 R_{n+1}(x)\,dx\right| \le \int_0^1 \frac{x^n}{(n+1)!} dx$ (+2) $= \boxed{\frac{1}{(n+1)!(n+1)}}$ (+1).

Bound

4. Consider the power series $\sum_{k=0}^{\infty} a_k (x-a)^k$. If this power series equals $f(x)$ for some $f \in C^{\infty}$, what are the values of the coefficients a_k in terms of f and its derivatives? Based on this power series, what power series represents $f'(x)$.

$a_k = \frac{f^{(k)}(a)}{k!}$ (+2)

Any Form (+3)

$f'(x) = \sum_{k=1}^{\infty} k a_k (x-a)^{k-1} = \sum_{k=1}^{\infty} \frac{f^{(k)}(a)}{(k-1)!} (x-a)^{k-1}$
$= \sum_{k=0}^{\infty} \frac{f^{(k+1)}(a)}{k!} (x-a)^k$