

Quiz 4

A Term, 2015

High: 20
Median: 18
Low: 15

(5 points each) For each series, please state whether it converges or diverges (circle one), and which test(s) you use to reach your conclusion. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

$$1. \sum_{n=1}^{\infty} \frac{1}{1+3^n}$$

Because $3^n < 3^n + 1$, $\frac{1}{3^n+1} < \frac{1}{3^n}$.
Since $\sum \frac{1}{3^n}$ is a convergent geometric series, $\sum \frac{1}{1+3^n}$ also converges by the Comparison Test.

or

Because $\lim_{n \rightarrow \infty} \frac{3^n}{1+3^n} = 1$, the Limit Comparison Test also implies $\sum \frac{1}{1+3^n}$ converges since $\sum \frac{1}{3^n}$ is a convergent geometric series.

converges diverges

$$2. \sum_{n=1}^{\infty} \frac{n}{4n^2+5}$$

Since $\int_1^{\infty} \frac{x dx}{4x^2+5} = \frac{1}{8} \int_9^{\infty} \frac{du}{u}$ diverges, by the integral test, this series must also diverge.

or

Because $\lim_{n \rightarrow \infty} \frac{\frac{n}{4n^2+5}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n^2}{4n^2+5} = \frac{1}{4}$, the Limit Comparison Test also implies this series diverges, since the harmonic series diverges.

converges diverges

$$3. \sum_{k=2}^{\infty} \frac{1}{k\sqrt{\ln k}}$$

Since $\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}} = \int_{\ln 2}^{\infty} u^{-1/2} du$ diverges, by the integral test, this series must also diverge.

Comparison Tests fail in this case.

converges diverges

$$4. \sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{\sqrt{\ln k}} = \sum_{K=2}^{\infty} (-1)^{K+1} a_K \text{ where } a_K = \frac{1}{\sqrt{\ln K}}$$

Since $a_K \downarrow 0$, this series converges by the alternating series test.

converges diverges