MA1033 Intro III **Name:**______________________________

Final **A Term, 2015**

This is a closed book/notes test. Show all work needed to reach your answers.

1. (36 points) Please determine whether each of the following converge or diverge; if possible, please give the value explicitly. Please name any test/series that you use.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{7^k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2 2^k}{k!}$

(c) $\displaystyle\int_0^2 \frac{dx}{2-x}$

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{1-2^{-k}}{1+k^2}$

2. (30 points) Please find the Taylor series with $x_0 = 0$ for each function; you may use the known series for common functions.

(a) $f(x) = \dfrac{x}{1 - x^3}$

Taylor Series: ______________________

(b) $g(x) = \displaystyle\int_0^x \cos(t^2)\, dt$

Taylor Series: ______________________

(c) $h(x) = (x + 1)e^{x+1}$

Taylor Series: ______________________

3. (24 points) For the series $\sum_{k=1}^{\infty} a_k$ and its partial sums $A_n := \sum_{k=1}^{n} a_k$ for any $n \in \mathbb{Z}^+$:

(a) What is the definition of *convergence* for this series?

(b) If this series converges, what must be true regarding the sequence $\{a_k\}$?

(c) Please complete the following proof of your answer to (b):

PROOF: Given any $\epsilon > 0$, since the series converges, $\exists\ L \in \mathbb{R}$ and $\exists\ N \in \mathbb{Z}^+$ such that

$$\left| \underline{\hspace{3cm}} - L \right| < \epsilon/2$$

whenever $\underline{\hspace{3cm}}$. But then by the triangle inequality,

$$|a_n| = |A_n - A_{n-1}| \leq |A_n - L| + \underline{\hspace{3cm}} \leq \underline{\hspace{3cm}}$$

whenever $\underline{\hspace{3cm}}$. Thus the sequence $\{a_n\}$ $\underline{\hspace{3cm}}$.

QED

4. (10 points) Suppose $\{f_n\}$ is a sequence of functions where $f_n : [0, +\infty) \to \mathbb{R}$ and

$$f_n(x) = \frac{x}{n} \exp\left(-\frac{x}{n}\right) .$$

Please explain why this sequence converges pointwise to zero, but is not uniformly convergent.